Nonlinear BEC Dynamics Induced by the Harmonic Modulation of the Atomic *s*-wave Scattering Length

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Motivation: In the recent experiment [1], a harmonic modulation of the atomic *s*-wave scattering length induces a nonlinear dynamics of a ^{7}Li BEC, and the resulting resonance curve for the excited quadrupole mode is measured. By combining a perturbative calculation with a numerical approach for solving the underlying Gross-Pitaevskii equation, we investigate in detail the frequency shift of a collective BEC mode which is due to nonlinear interaction effects [2].

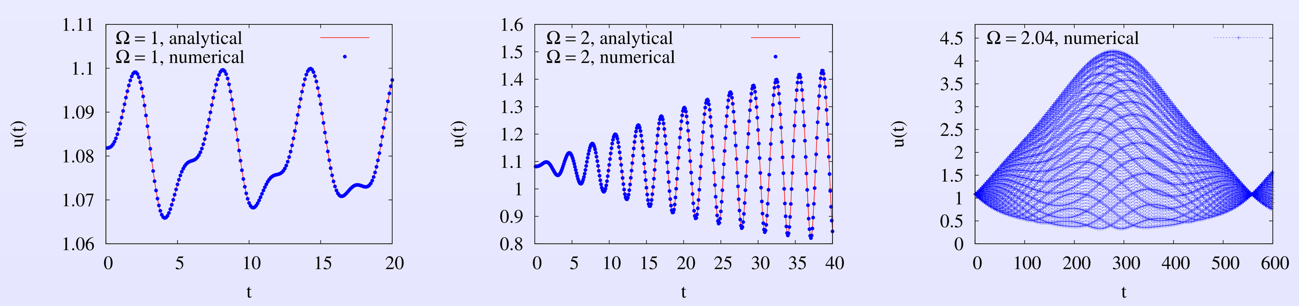
Time-dependent variational description of BEC

- Time-dependent GP equation can be studied using a Gaussian variational ansatz [3]. The dimensionless condensate width $u(t)$ evolves according to

$$\ddot{u}(t) + u(t) - \frac{1}{u(t)^3} - \frac{P}{u(t)^4} = 0\,,$$

where $P = \sqrt{\frac{2}{\pi}}\frac{Na}{l}$, a is the s-wave scattering length, $l = \sqrt{\frac{\hbar}{m\omega}}$ is the harmonic length scale, and N is the number of atoms.

- Using Feshbach resonances, harmonic modulation of the scattering length was achieved [1], yielding the time-dependent interaction $P(t) = p + q \sin \Omega t$.
- Real-time dynamics for $p = 0.4$, $q = 0.06$ and different frequencies Ω:



Analytical Poincaré-Lindstedt analysis

- Linearization of the variational equation yields for vanishing driving $q = 0$ zeroth order collective mode $\omega = \omega_0$ of oscillations around the time-independent solution u_0:

$$\omega_0 = \sqrt{1 + \frac{3}{u_0^4} + \frac{4p}{u_0^5}}\,, \qquad u_0 - \frac{p}{u_0^4} - \frac{1}{u_0^3} = 0\,.$$

- To calculate the collective mode to higher orders, we rescale time as $s = \omega t$:

$$\omega^2\,\ddot{u}(s) + u(s) - \frac{1}{u(s)^3} - \frac{p}{u(s)^4} - \frac{q}{u(s)^4}\sin\frac{\Omega s}{\omega} = 0\,.$$

- Far from resonances, we assume the following perturbative expansions in q:

$$u(s) = u_0 + q\,u_1(s) + q^2\,u_2(s) + q^3\,u_3(s) + \ldots\,,$$
$$\omega = \omega_0 + q\,\omega_1 + q^2\,\omega_2 + q^3\,\omega_3 + \ldots$$

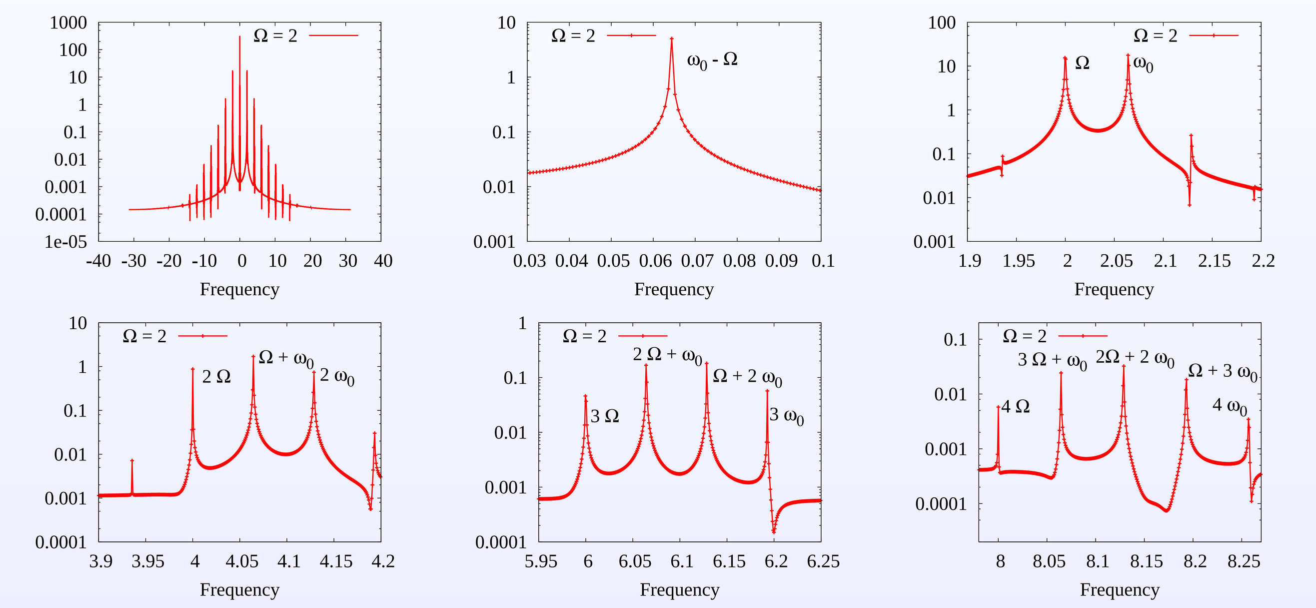
- This leads to a hierarchical system of equations in orders of q [4]:

$$\omega_0^2\,\ddot{u}_1(s) + \omega_0^2\,u_1(s) = \frac{1}{u_0^4}\sin\frac{\Omega s}{\omega}\,,$$
$$\omega_0^2\,\ddot{u}_2(s) + \omega_0^2\,u_2(s) = -2\omega_0\,\omega_1\,\ddot{u}_1(s) - \frac{4}{u_0^5}\,u_1(s)\,\sin\frac{\Omega s}{\omega} + \alpha\,u_1(s)^2\,,$$
$$\omega_0^2\,\ddot{u}_3(s) + \omega_0^2\,u_3(s) = -2\omega_0\,\omega_2\,\ddot{u}_1(s) - 2\beta\,u_1(s)^3 + 2\alpha\,u_1(s)u_2(s) - \omega_1^2\,\ddot{u}_1(s) + \frac{10}{u_0^6}\,u_1(s)^2\,\sin\frac{\Omega s}{\omega} - \frac{4}{u_0^5}\,u_2(s)\,\sin\frac{\Omega s}{\omega} - 2\omega_0\,\omega_1\,\ddot{u}_2(s),$$

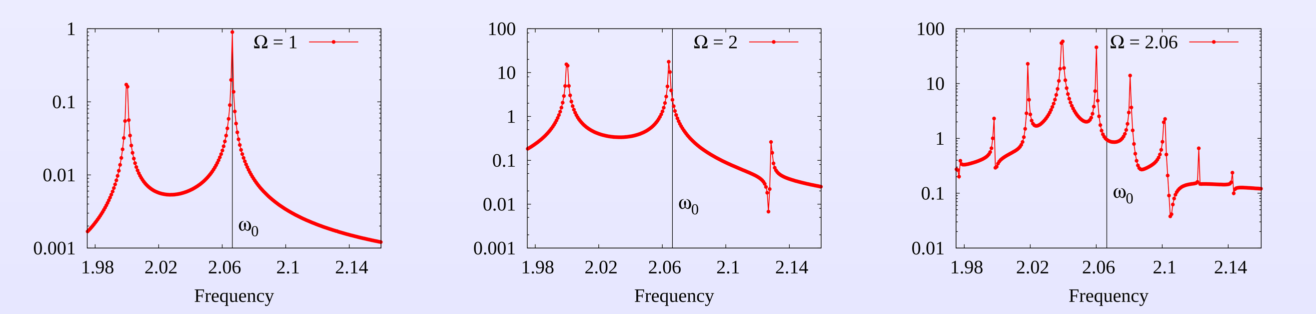
where $\alpha = 10p/u_0^6 + 6/u_0^5$ and $\beta = 10p/u_0^7 + 5/u_0^6$.

Fourier analysis of numerically obtained solutions

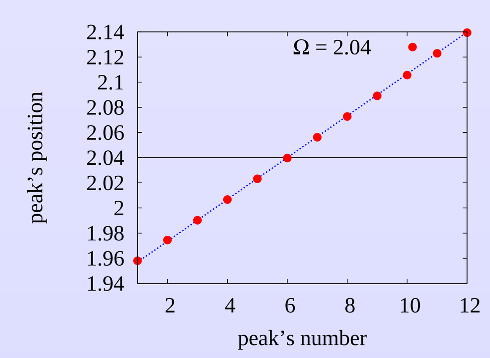
- The graphs give the Fourier spectrum of $u(t)$ for $p = 0.4$, $q = 0.06$ with basic modes $\omega_0 \approx 2.06638$ and Ω, higher harmonics $n\omega_0$ and $m\Omega$, as well as linear combinations $n\omega_0 + m\Omega$, including the beating frequency $|\Omega - \omega_0|$.



- Resonant effects are present for $\Omega \approx \omega_0$. Also, a complex peak structure close to ω_0 appears, and a shift in the frequency $\omega_0 \to \omega$ is clearly visible.



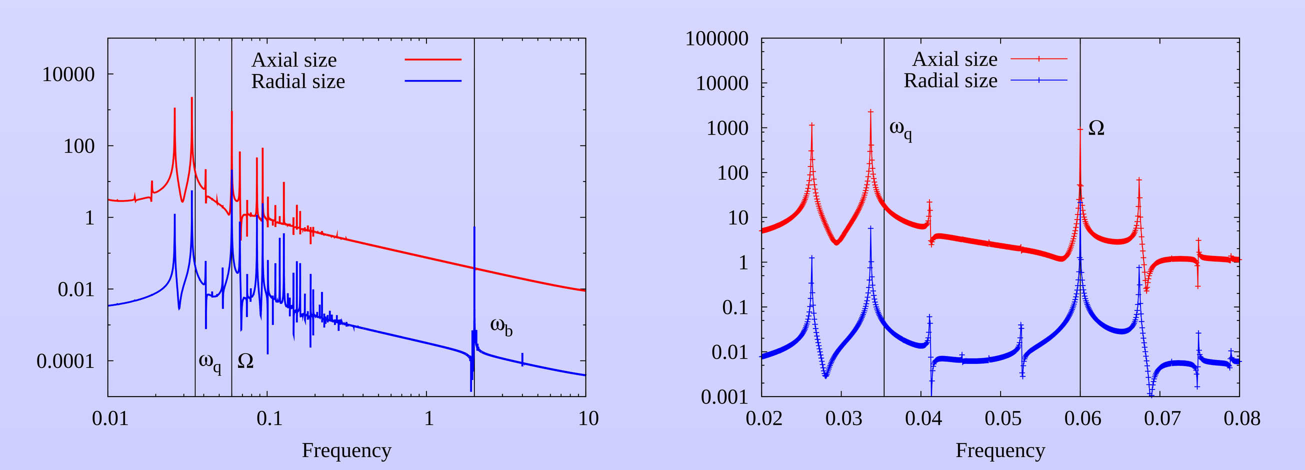
- Prominent peaks around ω_0 are equidistant, as we see from the graph for $q = 0.1$. By fitting $\omega(k) = A \times k + B$ to numerical data, we determine $A = |\omega - \Omega|$ for each Ω, and calculate $\omega = \Omega \pm A$, as given in the table.



Ω	A	$\Omega - A$	$\Omega + A$
2.00	0.0615	1.9352	**2.0609**
2.04	0.0166	**2.0232**	2.0562
2.05	0.0218	**2.0279**	2.0719
2.06	0.0273	**2.0326**	2.0876

Experimental setup

- Cylindrical trap with anisotropy $\lambda = 0.021$, $p = 15$, $q = 10$. Linear regime: quadrupole mode $\omega_q = 0.035375$, breathing mode $\omega_b = 2.00002$. Numerical results for driving frequency $\Omega = 0.06$:



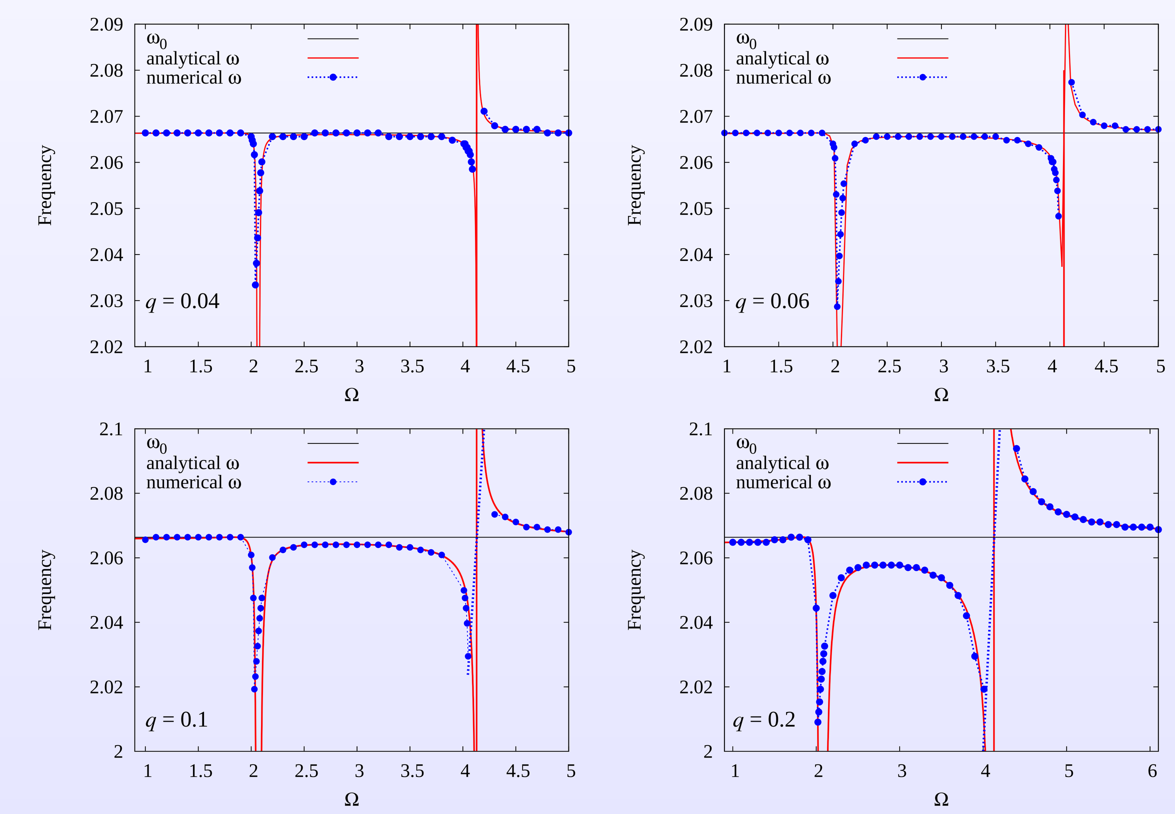
- **Result**: Frequency shift should be measurable in the experiment [1] for a sufficiently large driving amplitude q.

Frequency shift of the collective mode

- Frequency shift of the main collective mode is obtained using the third order Poincaré-Lindstedt method in q. First order correction ω_1 vanishes, leading to the frequency shift quadratic in q:

$$\omega = \omega_0 + q^2\,\frac{\text{Polynomial}(\Omega)}{(\Omega^2 - \omega_0^2)^2\,(\Omega^2 - 4\omega_0^2)}\,.$$

- Good agreement of numerical and analytical results is obtained for the frequency shift far from resonances for $p = 0.4$ and different q:



- The most significant shift of up to 5% is observed for $\Omega \approx \omega_0$ and large q.

Summary and outlook

- Using numerical Fourier analysis and analytical Poincaré-Lindstedt method, we calculated the frequency shift of the collective mode for a spherically symmetric BEC excited by harmonic modulation of the scattering length.
- In order to compare analytical results with the experiment [1], we are working on a similar perturbation theory for a cylindrically symmetric BEC.
- To further study nonlinear BEC dynamics effects, we will use numerical simulations of the full time-dependent Gross-Pitaevskii equation.

• References

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