Uticaj rasejanja na LO fononima i površinskim neravninama na rad kvantno kaskadnog lasera u jakom magnetnom polju

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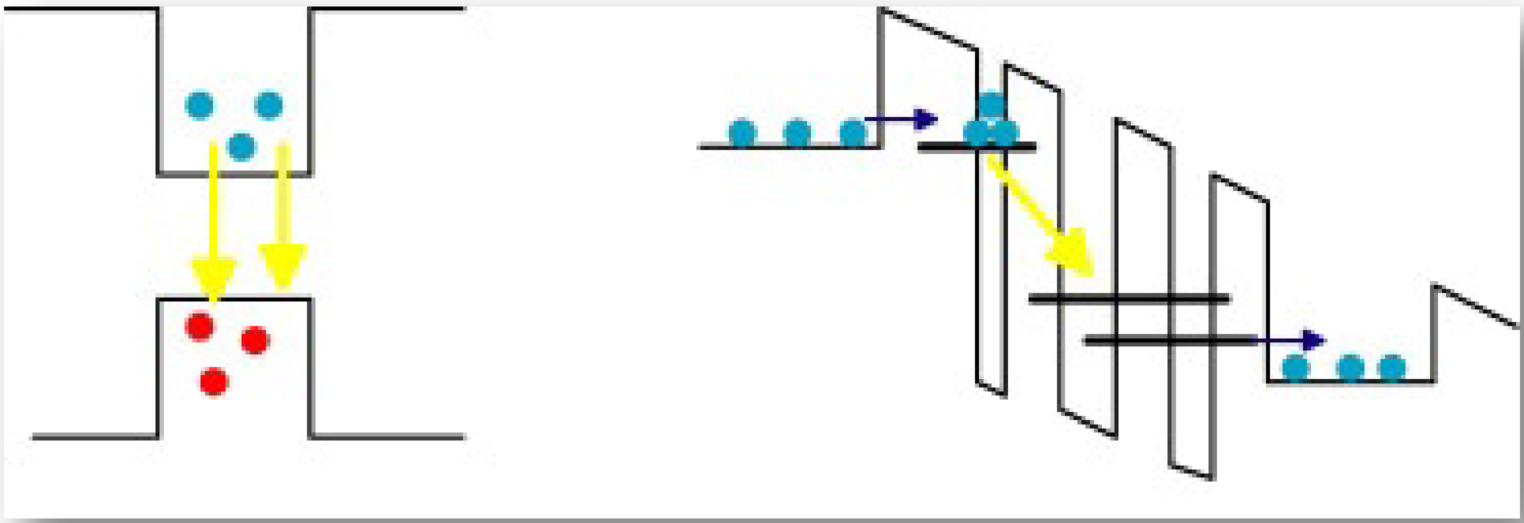
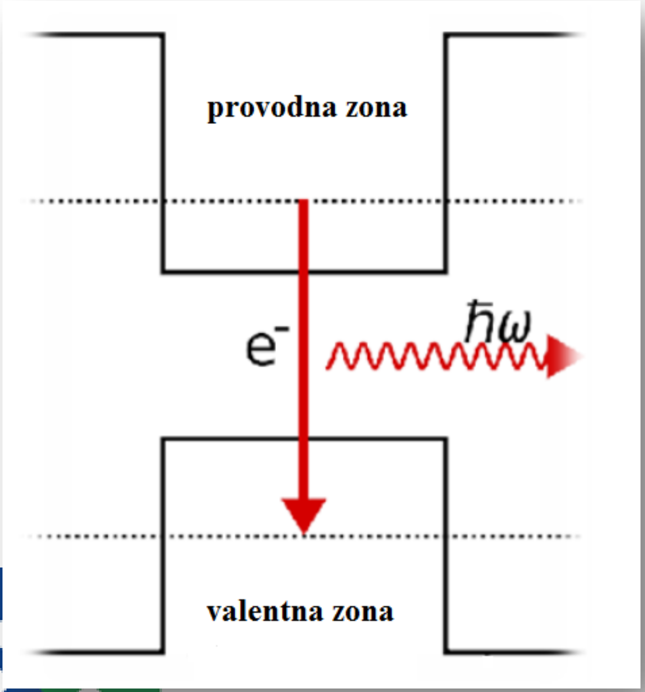
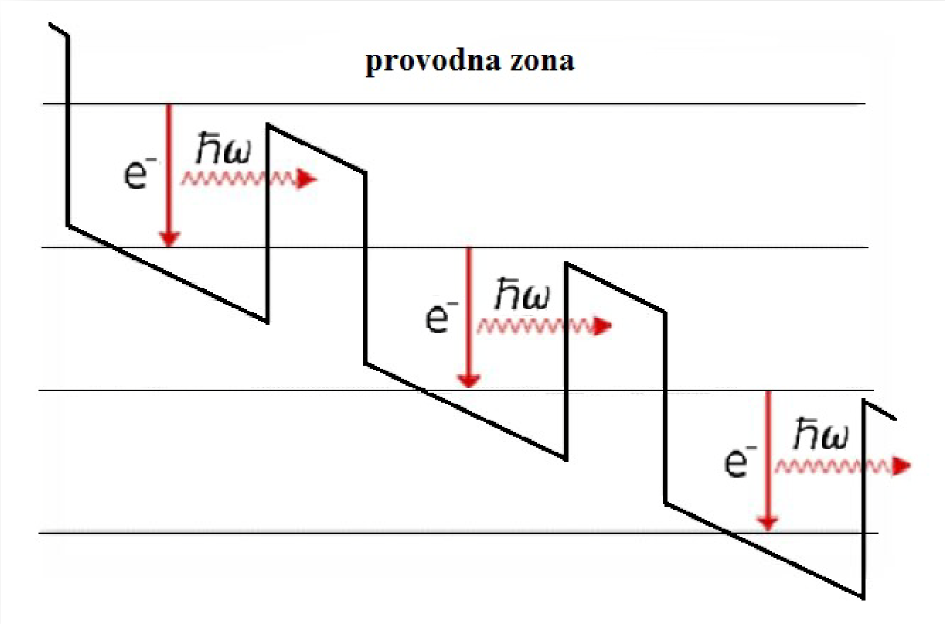
Ilustracija međuzonskog i unutarzonskog optičkog prelaza (1)



http://www.qoe.ethz.ch/research/topics/t-qcl

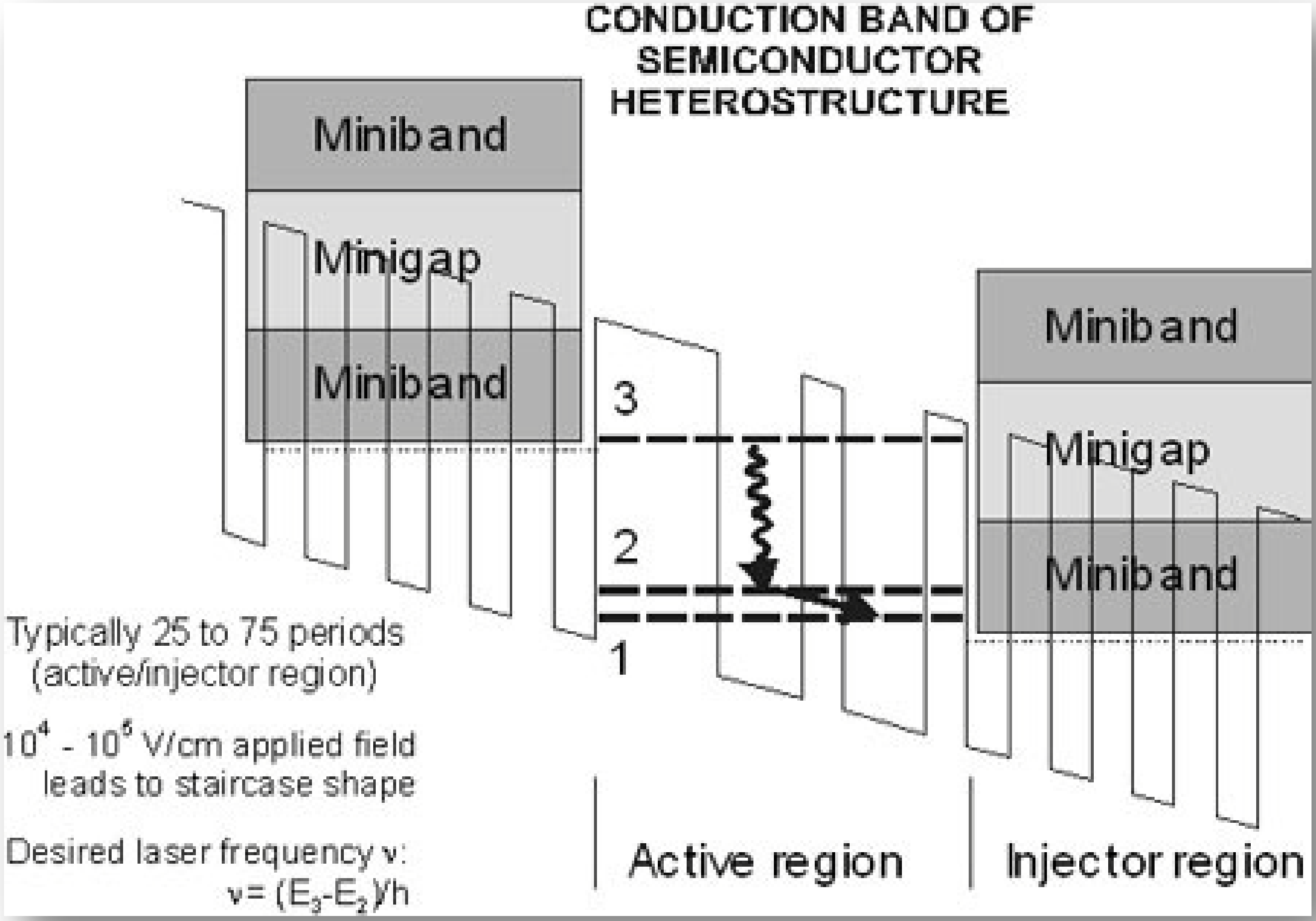
Ilustracija međuzonskog i unutarzonskog optičkog prelaza (2)





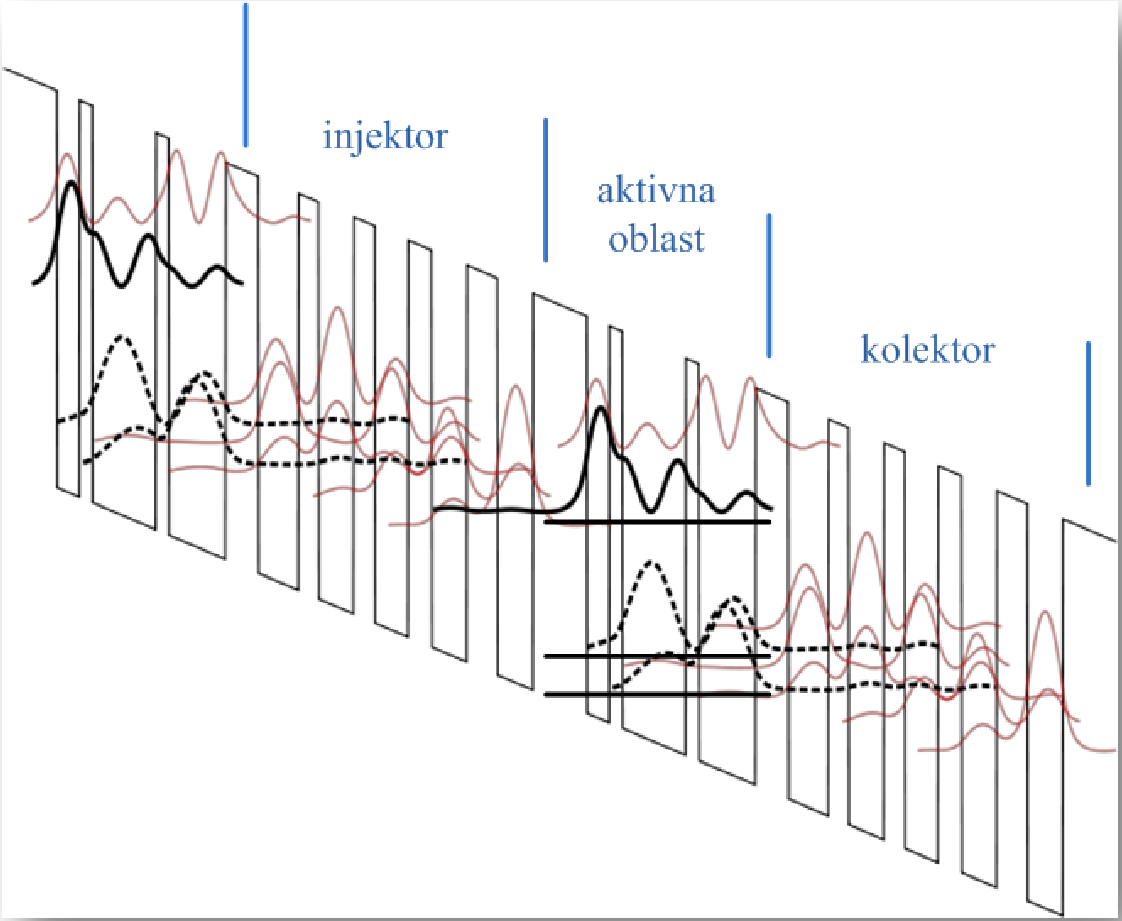
http://en.wikipedia.org/wiki/Quantum_cascade_laser

Kvantno kaskadni laser (1)



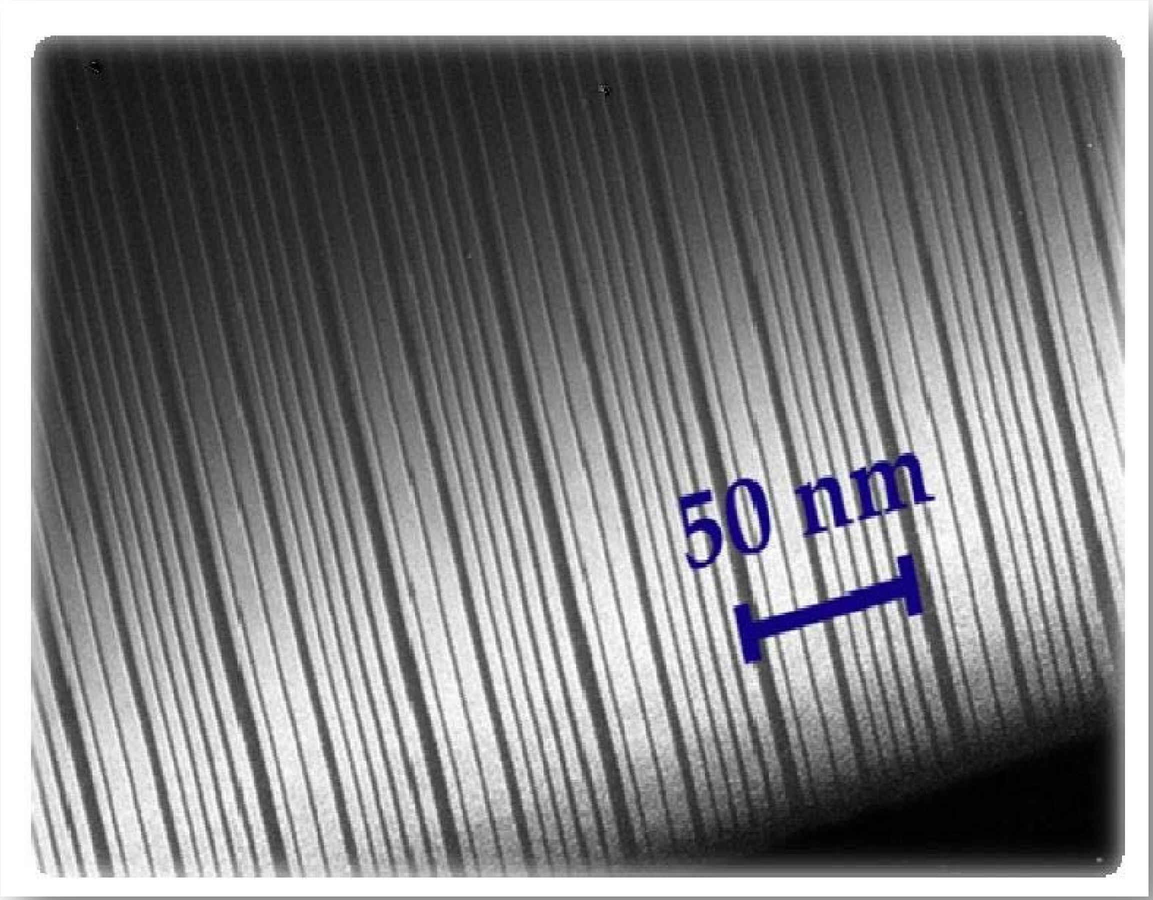
http://www.ru.nl/publish/pages/571693/qcl2.jpg

Kvantno kaskadni laser (2)



S. Ramović, Master rad

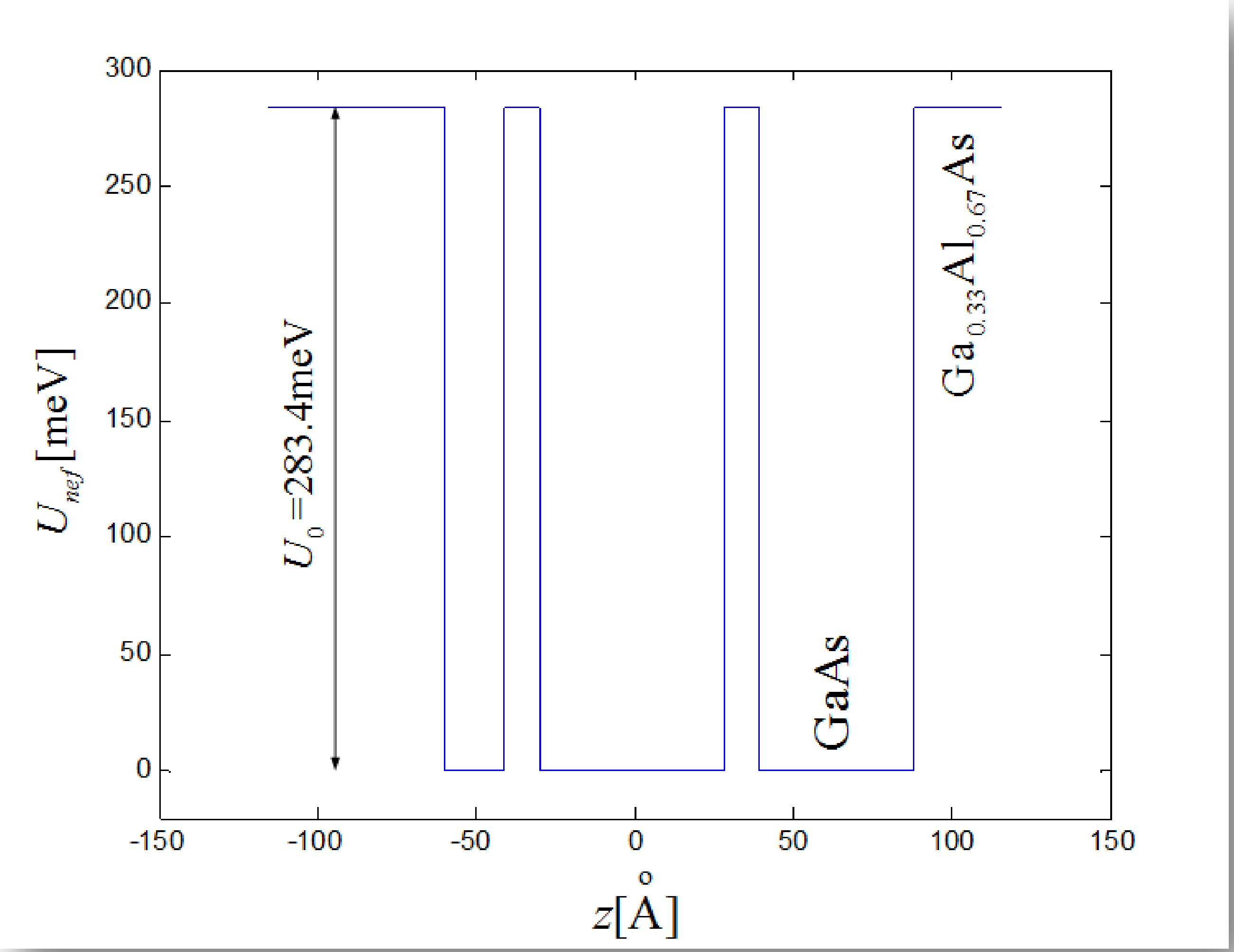
Prikaz periodične strukture kvantno kaskadnog



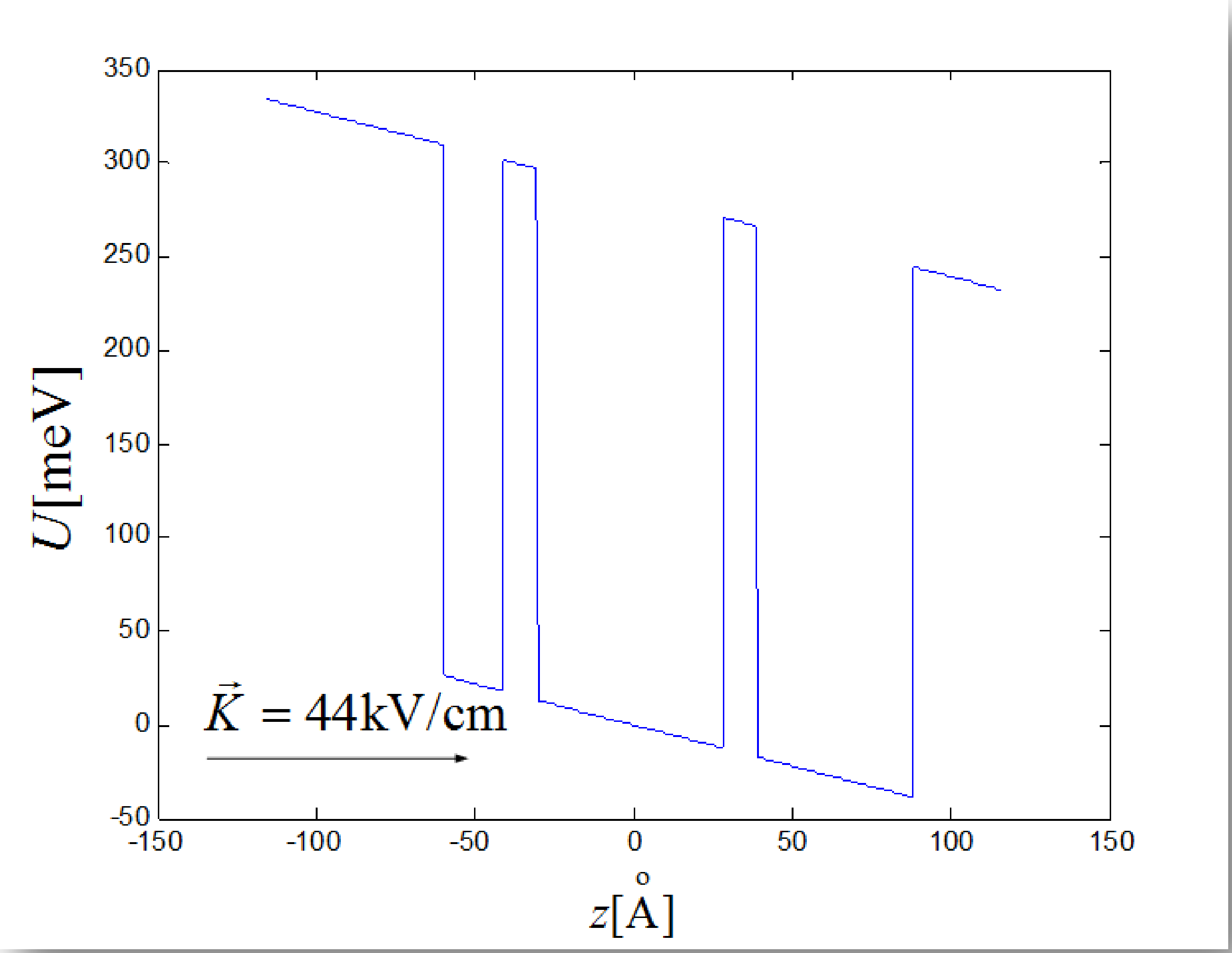
http://www.qoe.ethz.ch/mbe/Figures/TEMstructureS1549.jpg

Potencijal dna provodne zone bez prisustva spoljašnjeg polja - aktivna oblast

(D. Smirnov *et al.*, PRB 66, 125317 (2002) → dimenzije strukture)



Električno polje u smeru z - ose



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Šredingerova jednačina

$$-\frac{\hbar^2}{2}\frac{d}{dz}\left\{\frac{1}{m_\perp\left(z,E_n\right)}\left[\frac{d\eta\left(z\right)}{dz}\right]\right\}+U\left(z\right)\eta\left(z\right)=E_n\eta\left(z\right)$$

Efekat neparaboličnosti - longitudinalna masa

U.Ekenberg, PRB 40, 7714 (1989)

$$m_\perp(z,E_n)=\begin{cases}\left(m_{AlAs}X+m_{GaAs}\left(1-X\right)\right)\left(1+\frac{E_n-U_0}{E_{gb}}\right)\\ m_{GaAs}\left(1+\frac{E_n}{E_{gw}}\right)\end{cases}$$

$$m_{GaAs}=0.067m_0 \qquad E_{gw}=1.5041\text{eV}$$

$$m_{AlAs}=0.15m_0 \qquad E_{gb}=1.9624\text{eV}$$

$$X=0.33$$

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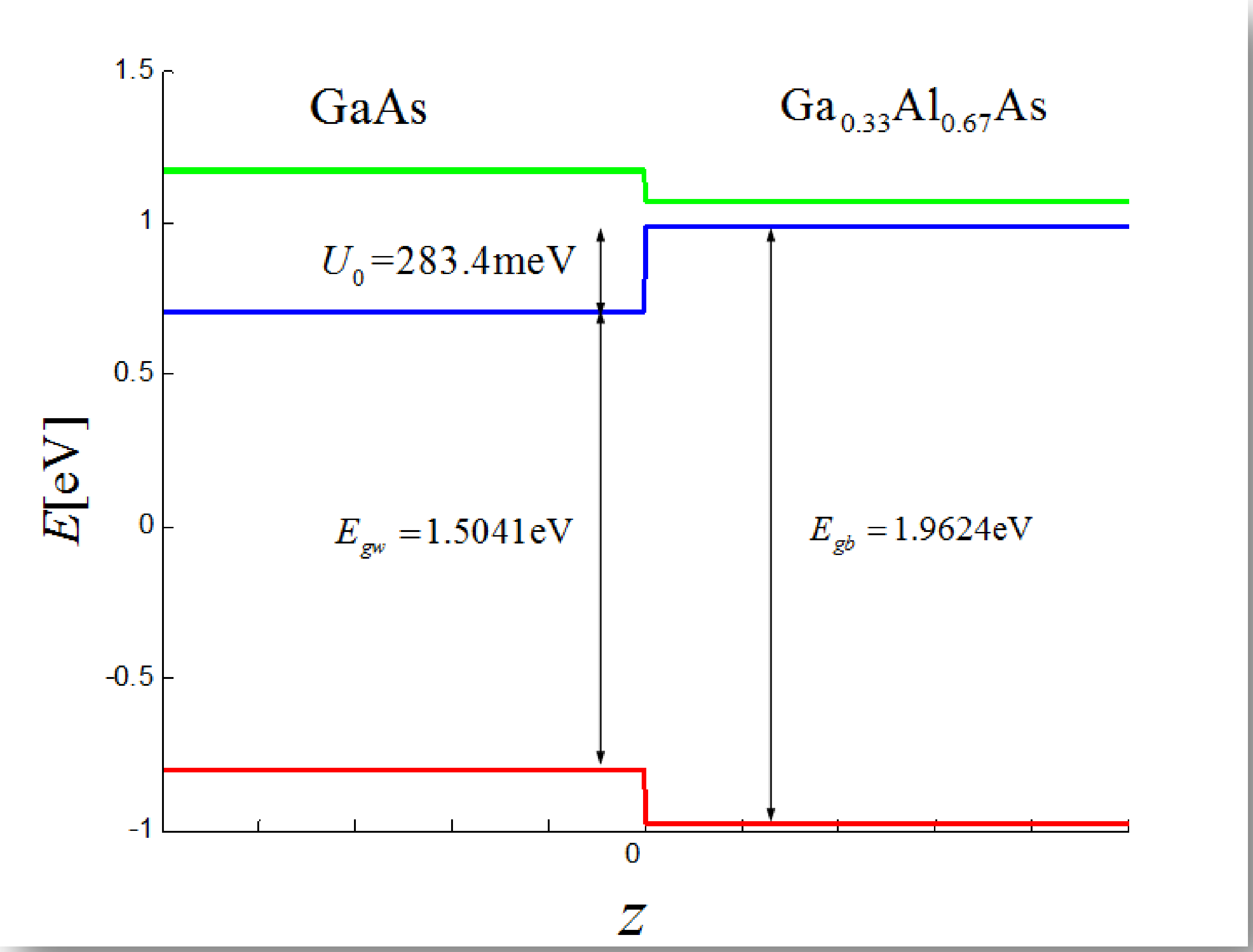
Struktura provodne i valentne zone



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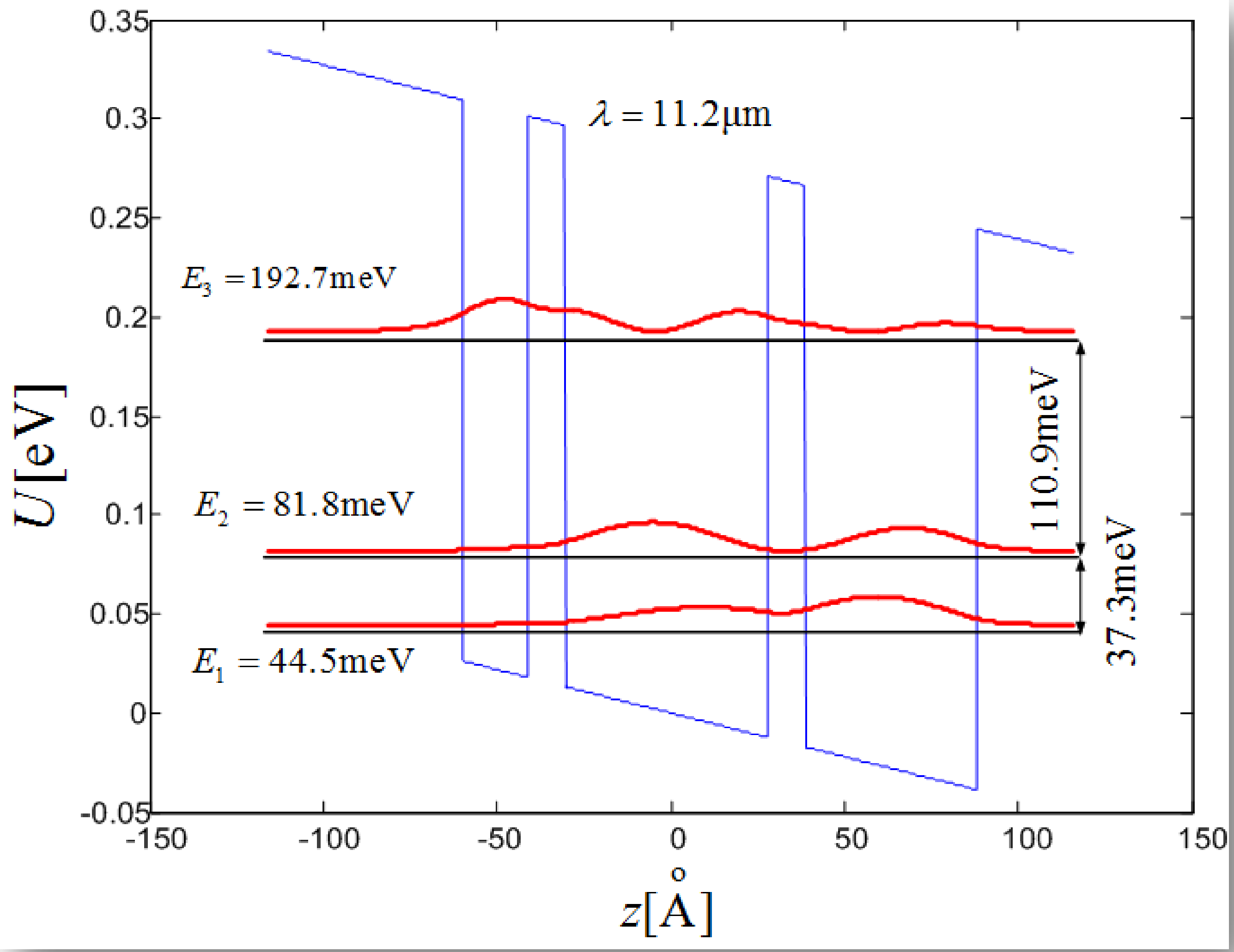
Sopstvene funkcije i energije dobijene *shooting* metodom



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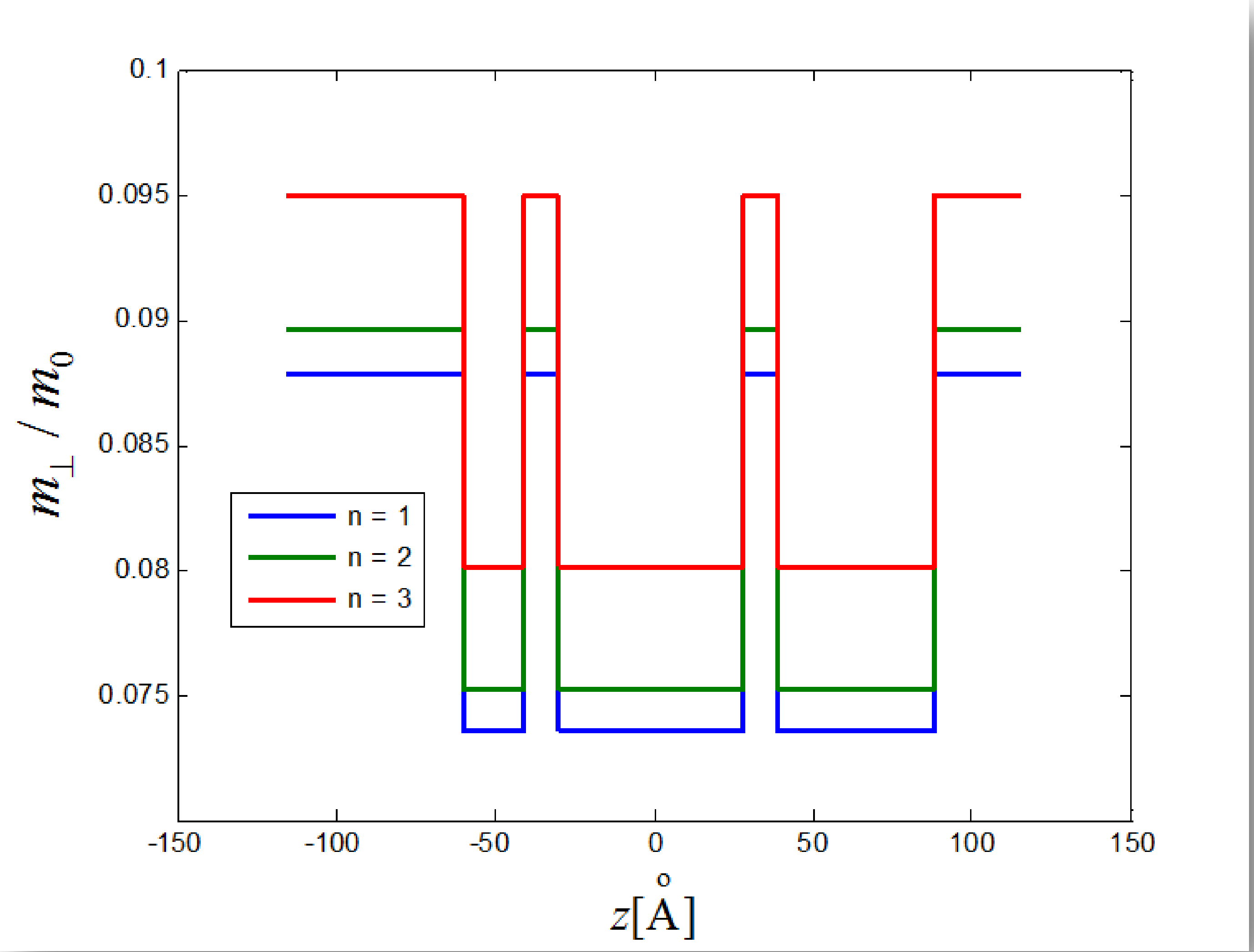
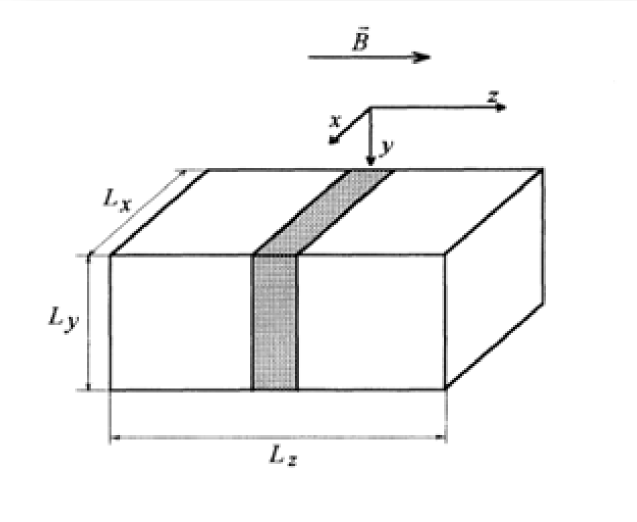
Efekat neparaboličnosti - longitudinalna masa



Magnetsko polje



$$\vec{B} = B\vec{e}_z$$

$$\vec{A}_M = -By\vec{e}_x$$

$$\psi(x, y, z) = \frac{e^{ik_x x}}{\sqrt{L_x}} \Phi_j(y - y_0)\eta(z)$$

$$y_0 = \hbar k_x / eB$$

$$-\frac{\hbar^2}{2}\frac{d}{dz}\left(\frac{1}{m_\perp(z,E)}\frac{d\eta(z)}{dz}\right) + \left(U(z) + \frac{eB\hbar}{m_\parallel(z,E)}\left(l + \frac{1}{2}\right)\right)\eta(z) = E\eta(z)$$

S. Živanović *et al.*, PRB 52, 8305 (1995)

Energije nakon primene magnetskog polja

$$E_{n,l}(B) = E_n + (l + 1/2)\frac{\hbar eB}{m_{\|}(E_n)} - \frac{1}{8}\left[\left(8l^2 + 8l + 5\right)\alpha_1^{'} + \left(l^2 + l + 1\right)\beta_1^{'}\right]\left(\frac{\hbar eB}{m_w}\right)^2$$

Efekat neparaboličnosti - transverzalna masa

U.Ekenberg, PRB 40, 7714 (1989)

$$\overline{m_{\|}^{-1}}(E_n) = \int \eta_n^2(z) m_{\|}^{-1}(z, E_n) dz$$

$$m_{\|}(z, E_n) = m_w \left\{1 + \left(2\alpha_1^{'} + \beta_1^{'}\right)\left[E_n - U_{nef}(z)\right]\right\}$$

$$\alpha_1^{'} = 0.642 \mathrm{eV}^{-1} \qquad \beta_1^{'} = 0.697 \mathrm{eV}^{-1}$$

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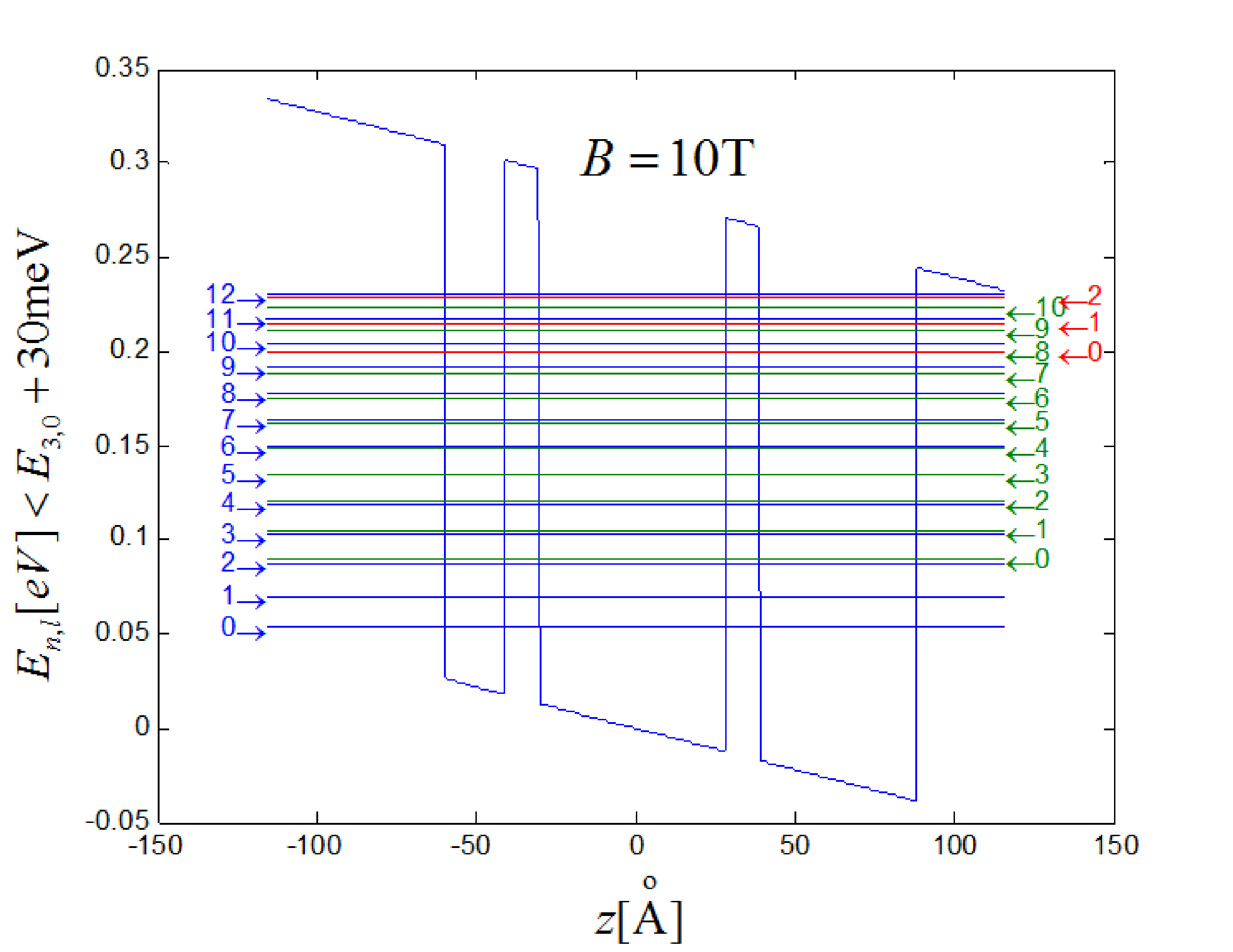
Energije za 10T



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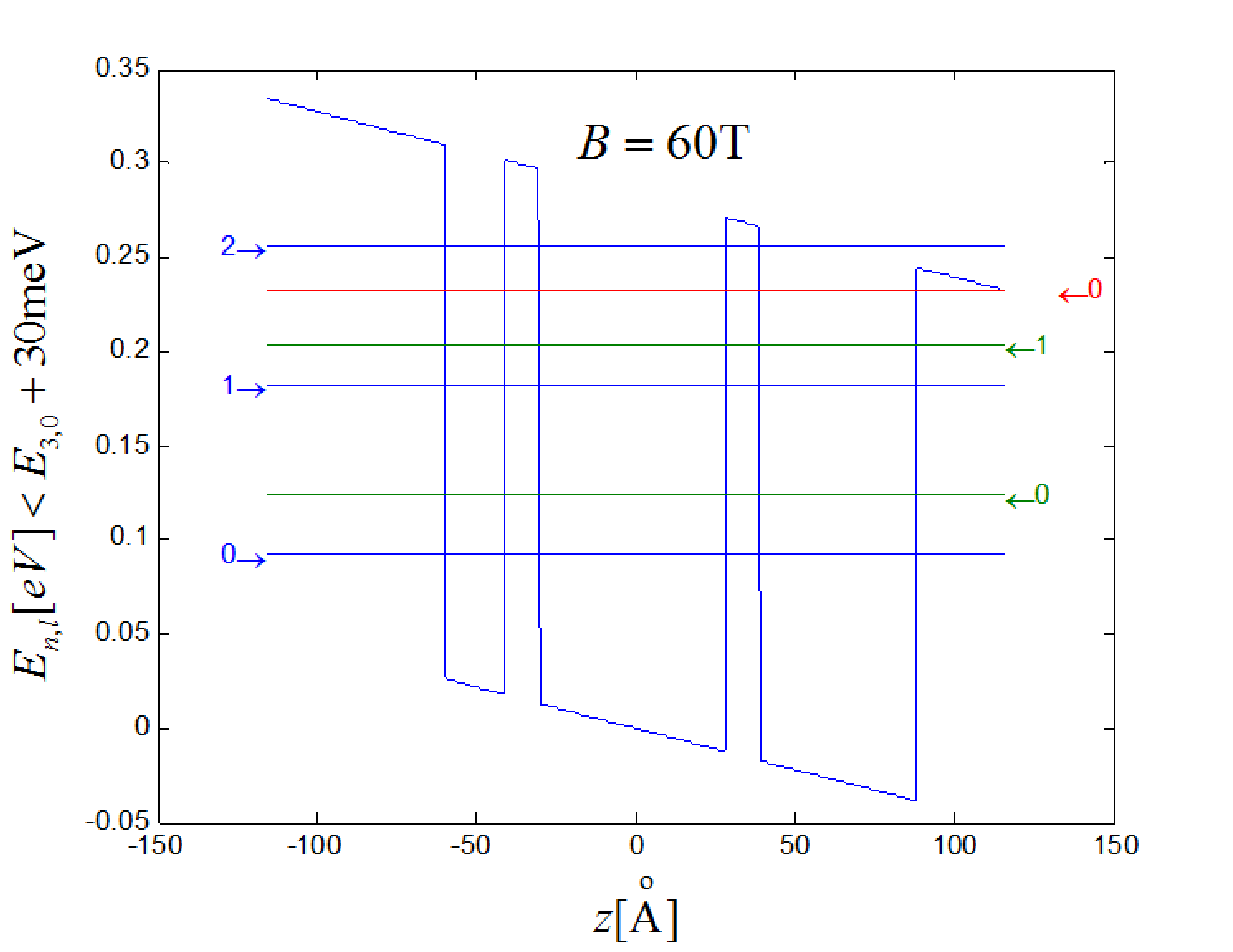
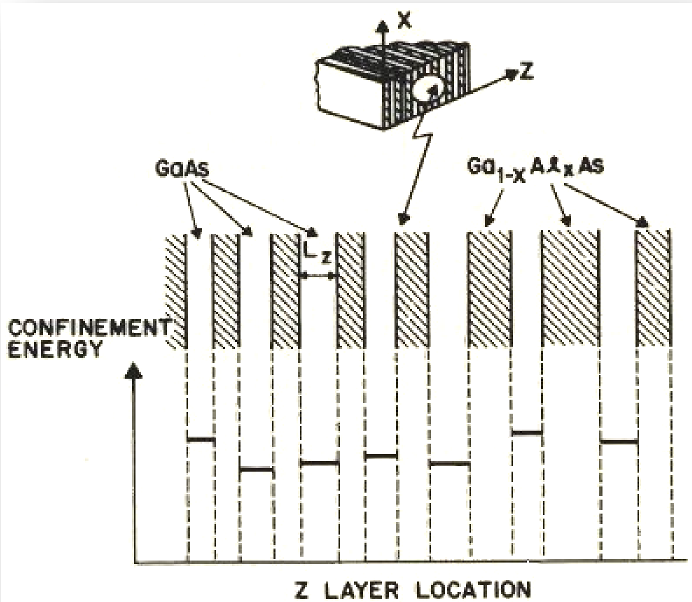
Energije za 60T



Širine jama imaju Gauss-ovu raspodelu

C. Weisbuch *et al*, SSC 38, 709 (1981)



$$\Pi(L_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(L - L_{i0})^2}{2\sigma^2}}$$

$$J = \int_{-\infty}^{+\infty} \Pi(L_i)\delta(E_{n_i,l_i} - E_{n_f,l_f} - \Delta E)dL_i$$

$$E_{n_i,l_i}(L_i) - E_{n_f,l_f}(L_i) \approx E_{n_i,l_i}(L_{i0}) - E_{n_f,l_f}(L_{i0}) - \gamma(L_i - L_{i0})$$

$$J = \frac{1}{\delta\sqrt{2\pi}} e^{\frac{\left(E_{n_i,l_i} - E_{n_f,l_f} - \Delta E\right)^2}{2\delta^2}}$$

$$\delta = \sigma\gamma = 6\text{meV}$$

C.Becker *et al*., PRB 69, 115328 (2004)

Elektrona - LO fonon interakcija

$$\hat{H}_{e-ph} = \sum_{\vec{q}} \hat{H}_{e-ph}(\vec{q}),$$

$$\hat{H}_{e-ph}(\vec{q}) = i\frac{g}{q}(e^{-i\vec{q}\cdot\vec{r}}a_{\vec{q}}^{+} - e^{i\vec{q}\cdot\vec{r}}a_{\vec{q}}),$$

g je Frölich faktor:

$$g^2 = 2\pi\hbar\omega_{LO}\frac{e^2}{\varepsilon_p V},$$

$$\varepsilon_p = \frac{4\pi\varepsilon_0}{\varepsilon_\infty^{-1} - \varepsilon_s^{-1}},$$

$$\hbar\omega_{LO} = 37.25\text{meV}$$
$$\varepsilon_\infty = 10.67$$
$$\varepsilon_s = 12.51$$

C.Becker *et al.*, PRB 69, 115328 (2004)

Brzina rasejanja elektron – LO fonon

(J. Radovanović et al., JAP 97, 103109 (2005) → zanemariti AC)

$$\frac{1}{\tau^{\mathrm{LO}}_{(n_i,l_i)\to(n_f,l_f)}} = \frac{2\pi}{\hbar}\sum_{\vec{q}}\left|\left\langle n_f, l_f, k_{x_f}, n_q \pm 1\right|\hat{H}_{e-ph}(\vec{q})\left|n_i, l_i, k_{x_i}, n_q\right\rangle\right|^2 J^{LO}$$

Emisija fonona:

$$\frac{1}{\tau^{\mathrm{LO,\{e\}}}_{(n_i,l_i)\to(n_f,l_f)}} = \frac{e^2\omega_{\mathrm{LO}}}{4\varepsilon_0}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_s}\right)\frac{1}{\delta\sqrt{2\pi}}e^{-\frac{\left(E_{n_i,l_i}-E_{n_f,l_f}-\hbar\omega_{\mathrm{LO}}\right)^2}{2\delta^2}}\left(n_q+1\right)\int_0^\infty\left|F(q_\parallel)\right|^2 G(q_\parallel)dq_\parallel$$

$$n_q = \frac{1}{e^{\frac{\hbar\omega_{\mathrm{LO}}}{k_B T}}-1} \qquad \left|F(q_\parallel, l_i, l_f)\right|^2 = e^{-\frac{q_\parallel^2}{2\beta^2}}\frac{l_i!}{l_f!}\left(\frac{q_\parallel^2}{2\beta}\right)^{l_f-l_i}\left[L_{l_i}^{l_f-l_i}\left(\frac{q_\parallel^2}{2\beta}\right)\right]^2, \beta = \sqrt{\frac{eB}{\hbar}}$$

$$G\left(q_\parallel\right) = \iint \eta_i^*(z)\eta_f(z)\eta_i(z')\eta_f^*(z')e^{-q_\parallel|z-z'|}\,dz\,dz'$$

C.Becker *et al.*, PRB 69, 115328 (2004) i V. Milanović, izvođenje

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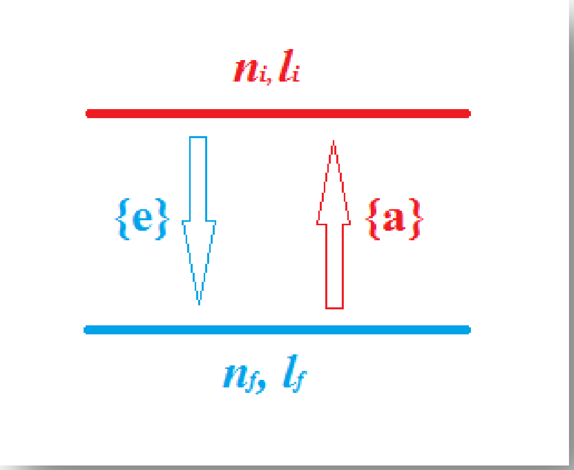
Apsorpcija fonona

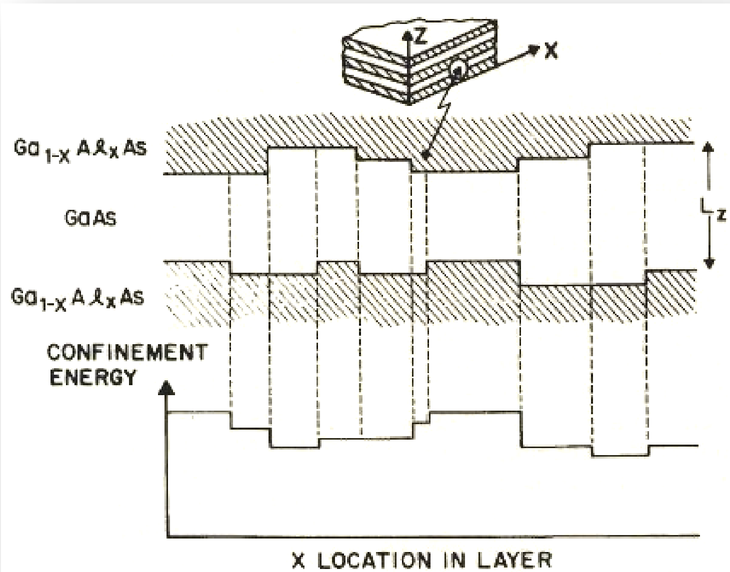
$$\frac{1}{\tau^{\mathrm{LO},\{a\}}_{(n_f,l_f)\to(n_i,l_i)}} = \frac{1}{\tau^{\mathrm{LO},\{e\}}_{(n_i,l_i)\to(n_f,l_f)}} \frac{1}{e^{\frac{\hbar\omega_{LO}}{kT}}}$$

$$e^{\frac{\hbar\omega_{LO}}{kT}} \approx \begin{cases} 200, T = 77\mathrm{K} \\ 4 \;\; , T = 300\mathrm{K} \end{cases}$$

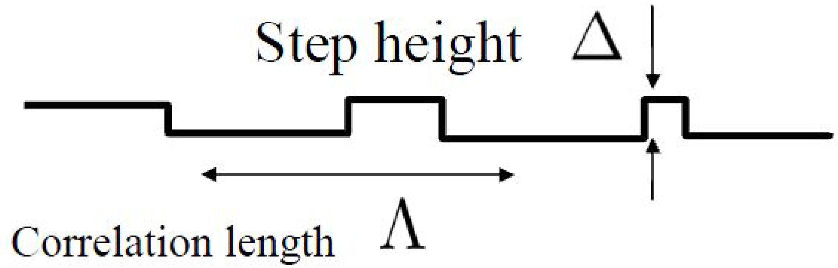
Zanemariti?



Interface roughness



C. Weisbuch *et al*, SSC 38, 709 (1981)



$\Delta = 1.5\,\overset{o}{A} \qquad \Lambda = 60\,\overset{o}{A}$

Gauss-ova korelaciona funkcija

T. Unuma et al., JAP, 93, 1586 (2003)

$$\langle \Delta(\vec{r})\Delta(\vec{r}\,') \rangle = \Delta^2 e^{-\frac{|\vec{r}-\vec{r}\,'|}{\Lambda^2}}$$

Brzina rasejanja na površinskim neravninima

$$\left\langle \frac{1}{\tau^{IR}_{(n_i,l_i)\to(n_f,l_f)}}(z_i) \right\rangle = \frac{2\pi}{\hbar} \left\langle \sum_{k_{x_i},k_{x_f}} \left\langle \left| \left\langle n_f, l_f, k_{x_f} \mid \hat{H}_{IR} \mid n_i, l_i, k_{x_i} \right\rangle \right|^2 \right\rangle \right\rangle J^{IR}$$

$$\hat{H}_{IR} = U_0 \delta(z - z_i) \Delta(x, y) \qquad J^{IR} = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{\left(E_{n_i,l_i} - E_{n_f,l_f}\right)^2}{2\delta^2}}$$

$$\frac{1}{\tau^{IR}_{(n_i,l_i)\to(n_f,l_f)}} = \sum_{z_i} \left\langle \frac{1}{\tau^{IR}_{(n_i,l_i)\to(n_f,l_f)}}(z_i) \right\rangle$$

$$\frac{1}{\tau^{IR}_{(n_i,l_i)\to(n_f,l_f)}} = \frac{1}{\tau^{IR}_{(n_f,l_f)\to(n_i,l_i)}}$$

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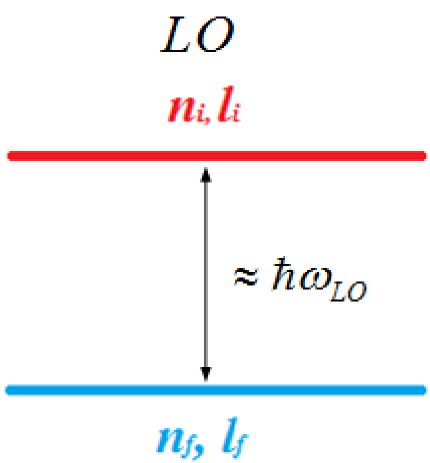
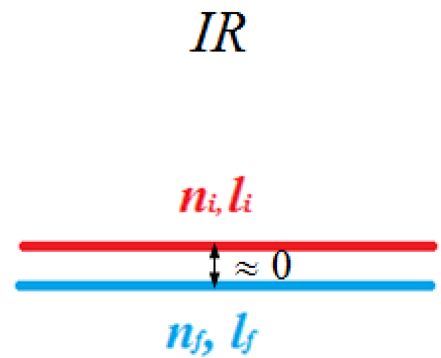
T. Unuma *et al.*, JAP 93, 1586 (2003) i V. Milanović, izvođenje

LO vs. IR

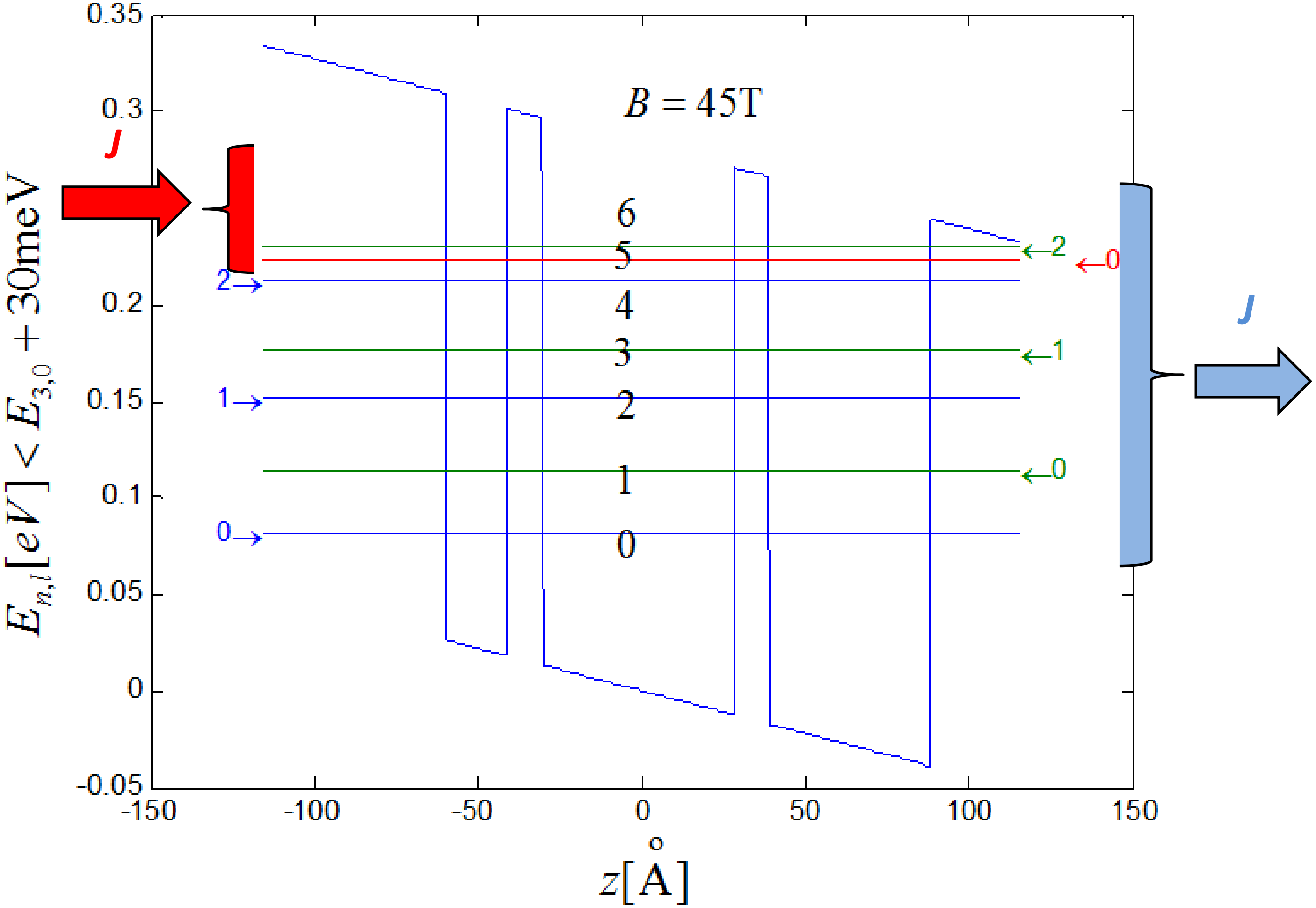
$$J^{LO} = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{\left(E_{n_i,l_i} - E_{n_f,l_f} - \hbar\omega_{\mathrm{LO}}\right)^2}{2\delta^2}}$$

$$J^{IR} = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{\left(E_{n_i,l_i} - E_{n_f,l_f}\right)^2}{2\delta^2}}$$





Struje


J
B = 45T
6
5
4
3
2
1
0
2→
1→
0→
←2
←0
←1
←0
J
E_{n,l}[eV] < E_{3,0} + 30meV
0.35
0.3
0.2
0.15
0.1
0.05
0
-0.05
-150
-100
-50
0
50
100
150
z[Å]

Brzinske jednačine

$$N_i \sum_{j \neq i} \frac{\bar{f}(E_j)}{\tau_{i \to j}} - \bar{f}(E_i) \sum_{j \neq i} \frac{N_j}{\tau_{j \to i}} + \frac{J_i}{e} = 0$$

$$\bar{f}(E_k) = 1 - \frac{\pi \hbar}{eB} N_k$$

$$J_i = \begin{cases} -J \dfrac{e^{-E_{3,i}/k_B T}}{\sum_k e^{-E_{3,k}/k_B T}} \\ J \dfrac{e^{-E_{1,i}/k_B T}}{\sum_k e^{-E_{1,k}/k_B T}} \end{cases}$$

J. Radovanović et al., JAP 97, 103109 (2005)

Nelinearne, iterativno ...

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Optičko pojačanje

$$g_{3\to 2} = \frac{2e^2\pi^2}{\bar{n}\varepsilon_0}\frac{d^2_{3\to 2}}{\lambda}\sum_i \delta\left(E_{3,i} - E_{2,i} - \hbar\omega\right)\left(N_{3,i} - N_{2,i}\right)$$

$$\bar{n} = 3.3$$

$$d_{3\to 2} = \int \eta_3 z \eta_2 dz + \frac{\hbar^2}{2\left(E_3 - E_2\right)}\int \eta_3 \frac{d}{dz}\left\{\left(\frac{1}{m_2} - \frac{1}{m_3}\right) z \frac{d\eta_2}{dz}\right\} dz$$

$$g = \frac{g_{3\to 2}}{J}$$

V. Milanović izvođenje

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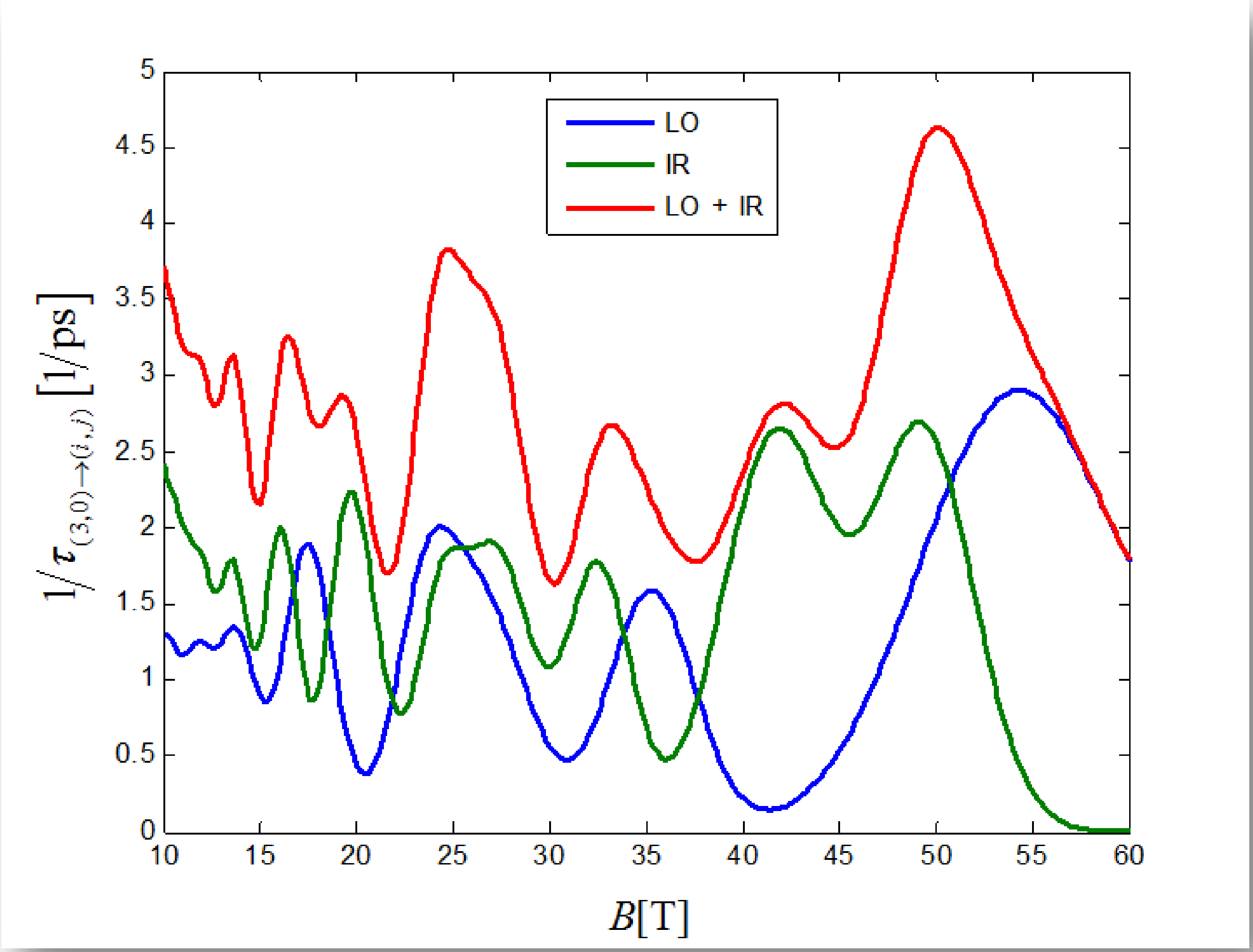
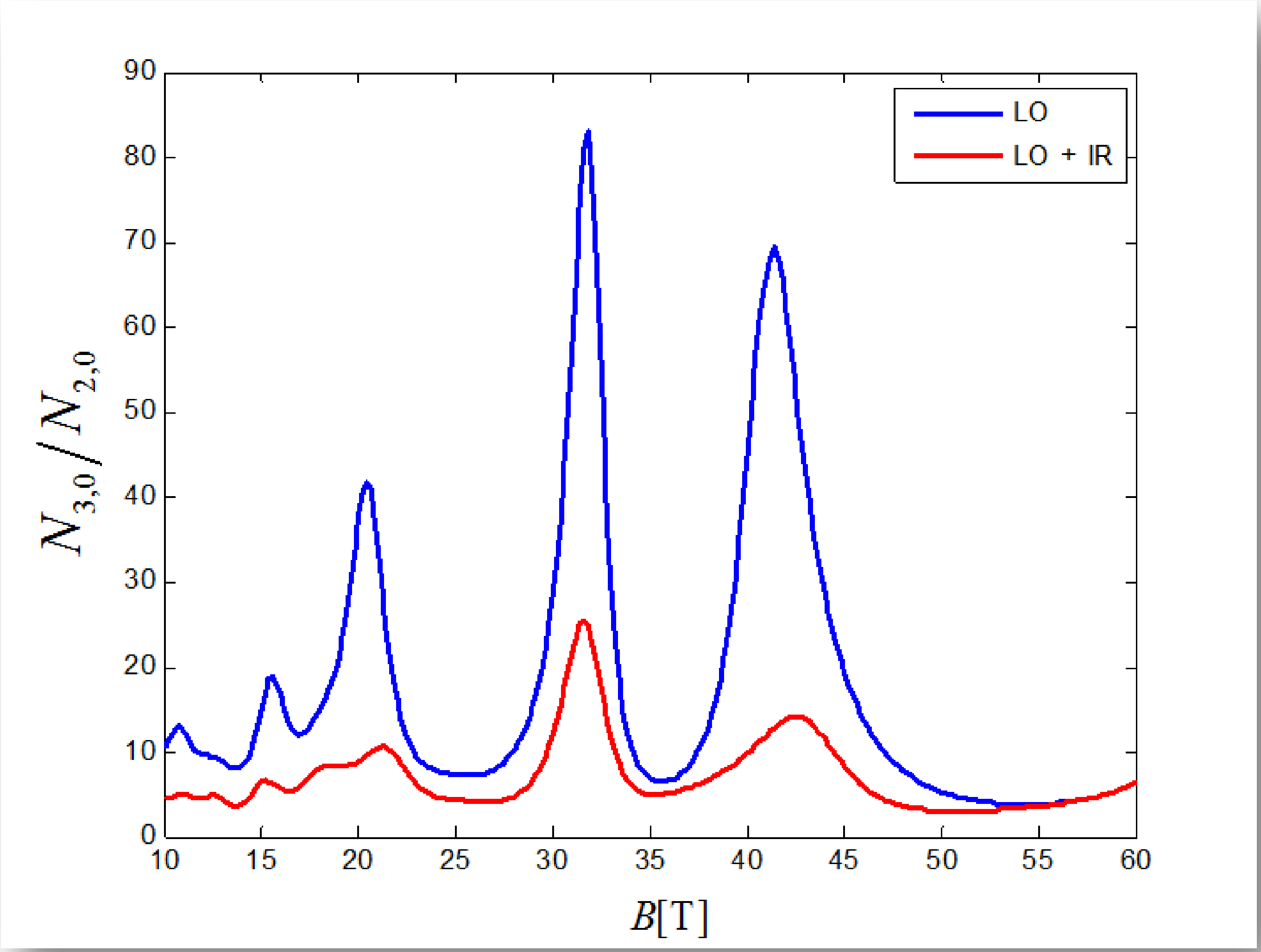
Brzina verovatnoće prelaza sa nivoa (3, 0) na ostale nivoe



Odnos povrsinskih koncentracija elektrona na nivoima (3, 0) i (2, 0)



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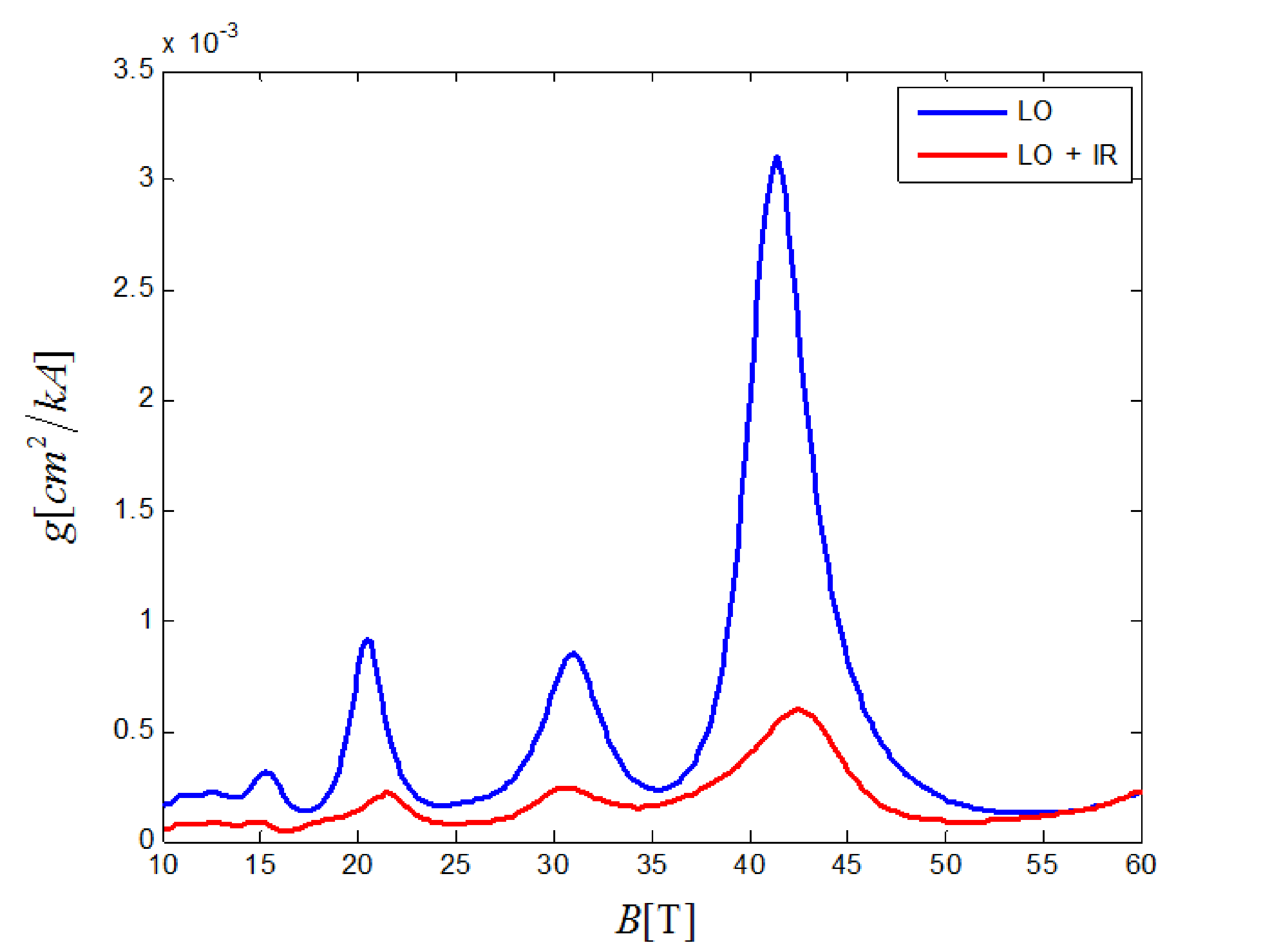
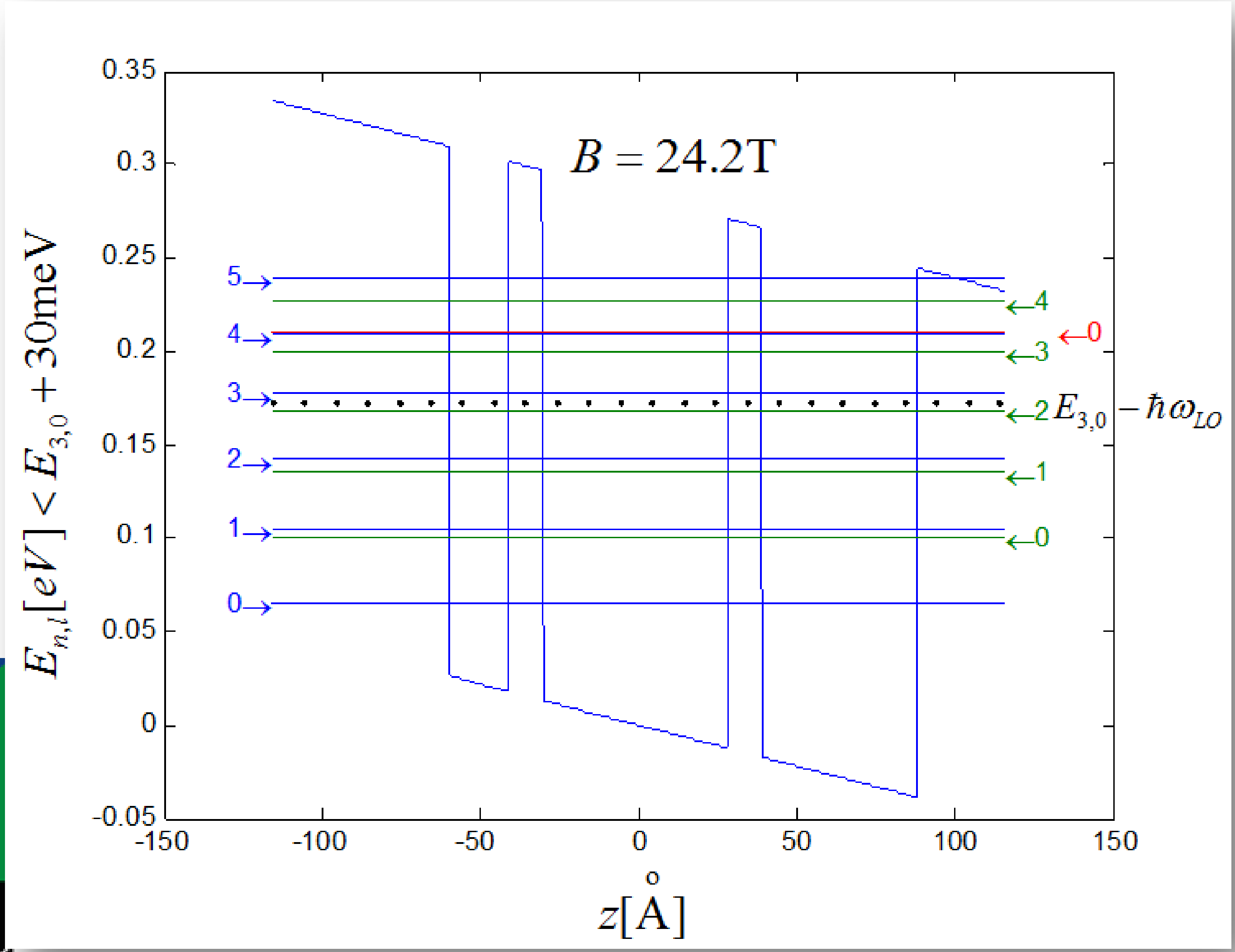
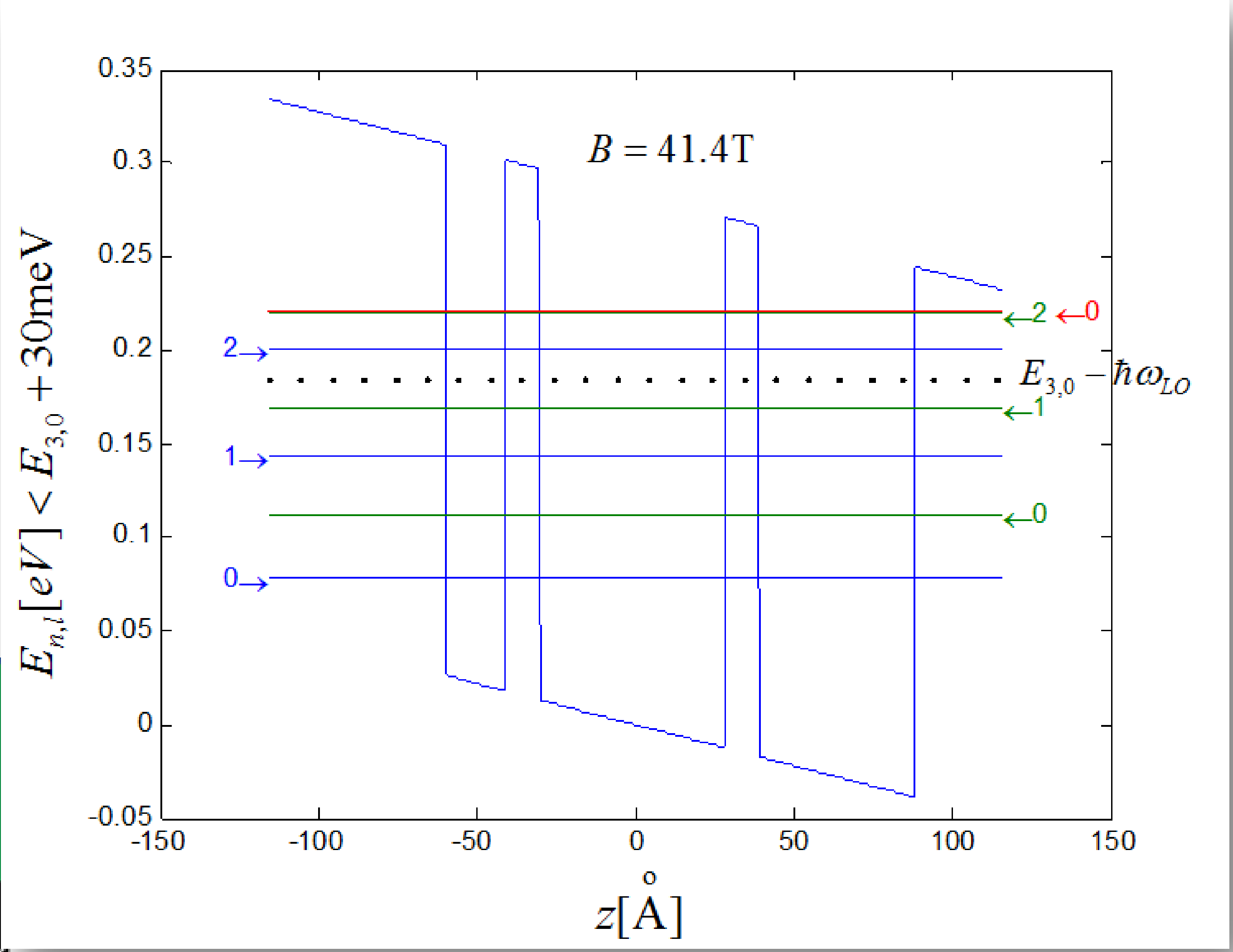
Optičko pojačanje QCL-a



Lokalni minimum pojačanja



Lokalni maksimum pojačanja



Matlab, Nehalem, 12 min, *B* < 10T
PC < 1h

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