

Marija Mitrović

Introduction
and
motivation-
connecting
structure and
dynamics

Model of
multiscale
(community)
networks

Spectral
properties

Conclusion

Modularity of networks from the perspective of spectral analysis

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Outline

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- 1 Introduction and motivation-connecting structure and dynamics
- 2 Model of multiscale (community) networks
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Multiscale structure of networks

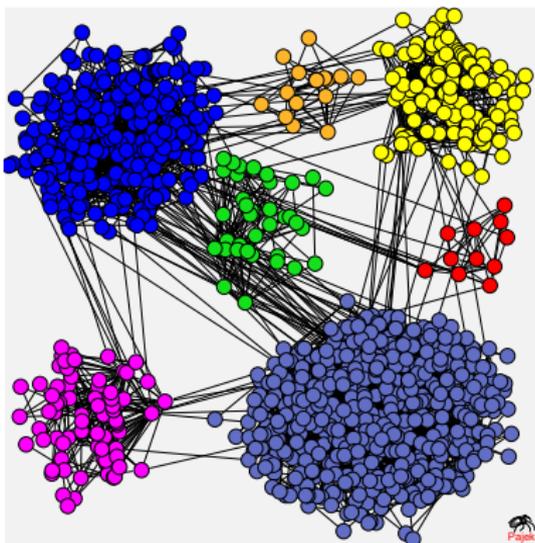
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- Modular structure - common properties of biological, social and IT networks.
- Connection between dynamics and structure of network.

Network of networks

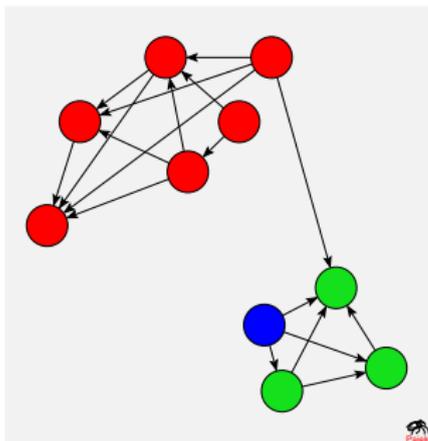
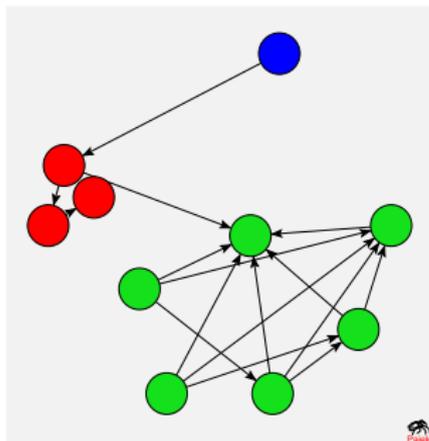
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Model parameters α , P_0 and M .

B. Tadić, Physica A 293, (2001).

M. Mitrović and B. Tadić, LNCS, (2008)

Multiscale networks

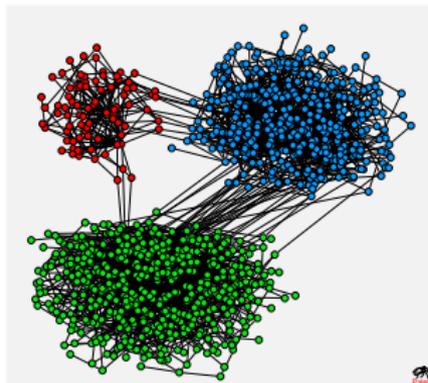
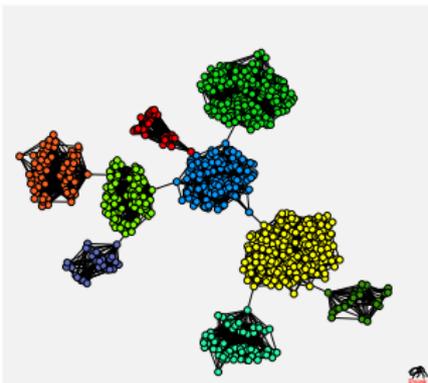
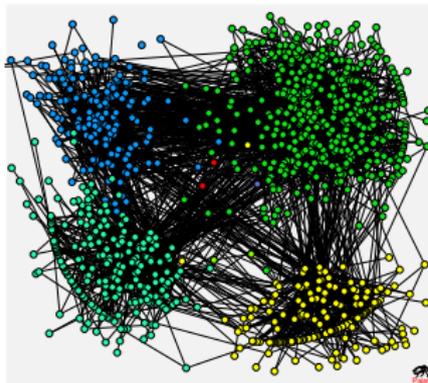
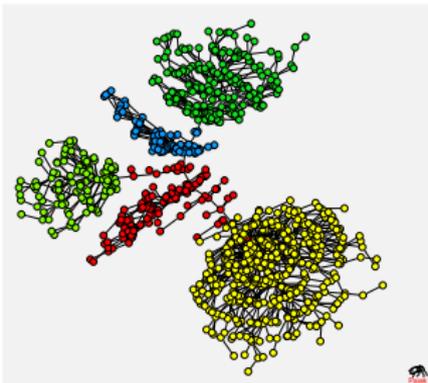
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Adjacency matrix

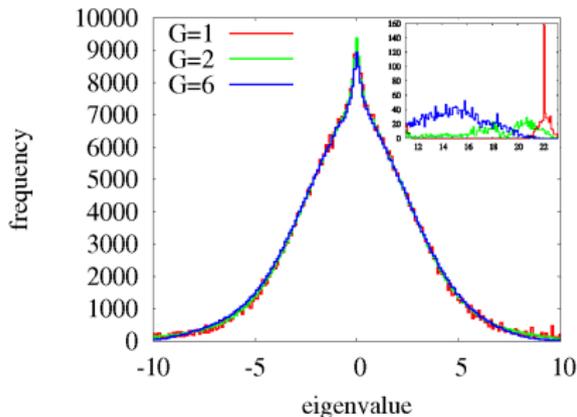
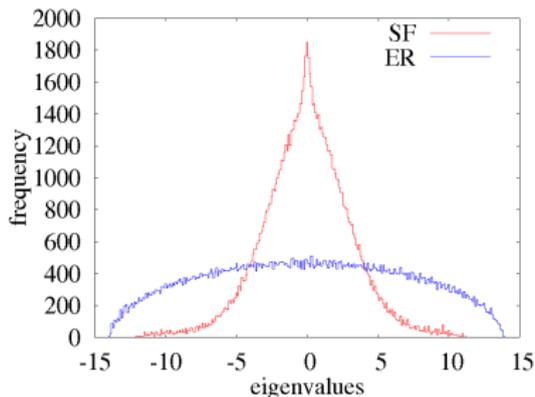
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Closer look at cliques on random tree

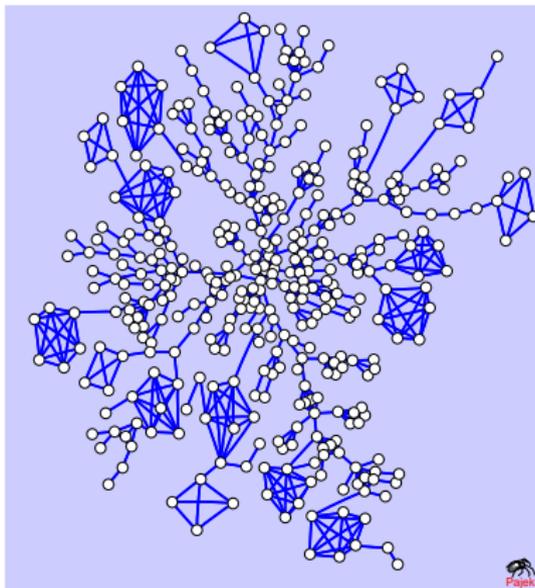
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- Clique of size N has one eigenvalue equal $\lambda_N = N - 1$ and $\lambda_1 = \dots = \lambda_{N-1} = -1$.
- Spectra of random tree with cliques contains information about number of cliques.

Spectral density of adjacency matrix

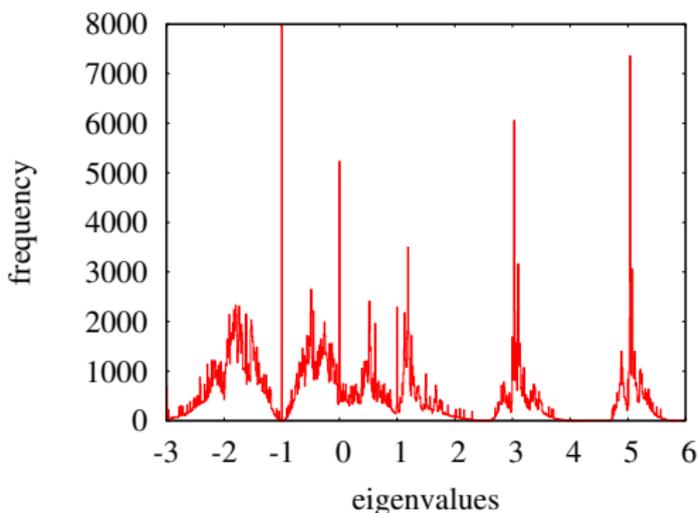
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Eigenvalues around $\lambda = 3$ and $\lambda = 5$ are related to existence of cliques of size $N = 4$ and $N = 6$.

Randomized networks

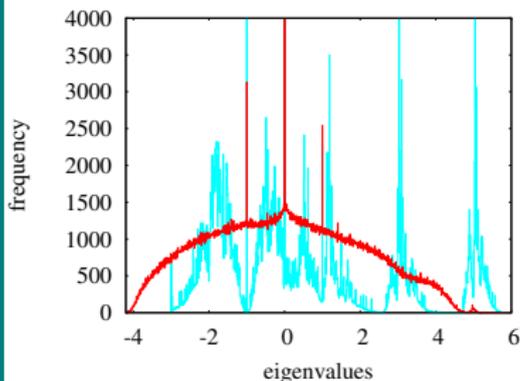
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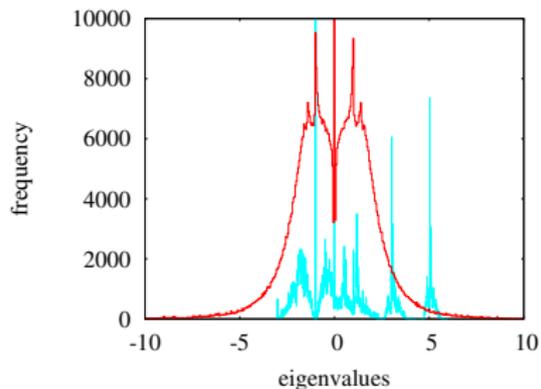
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Same degree distribution
and average connectivity.



Power law degree distribution
and same average
connectivity.

Eigenvectors

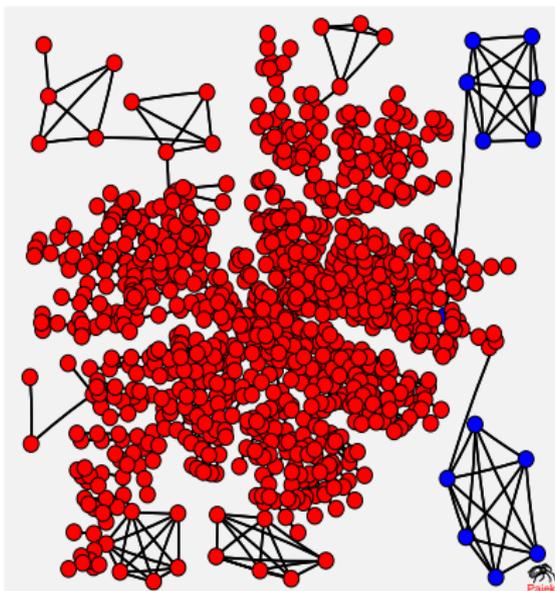
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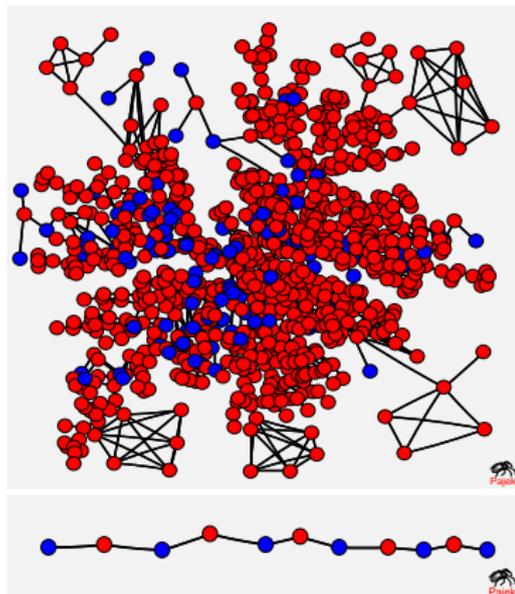
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Eigenvector for eigenvalue $\lambda = 5.04$ is localized on cliques.



Eigenvalue $\lambda = 0$ is related to chain with odd number of nodes.

Looking at the dynamics-random walk

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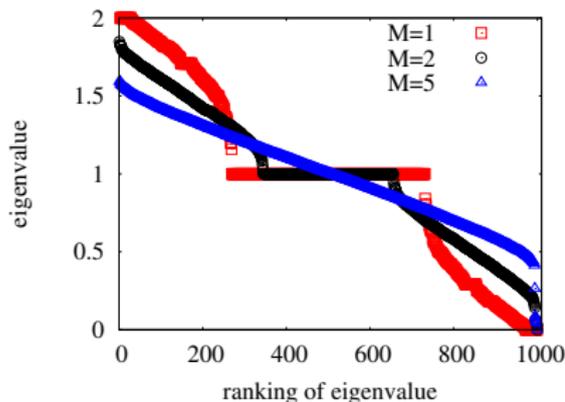
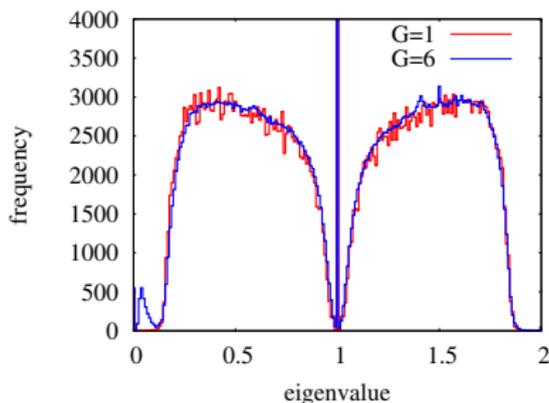
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Laplacian related to random walk
dynamics $L_{ij} = \delta_{ij} - \frac{1}{\sqrt{q_i q_j}} A_{ij}$

Ranking of eigenvalues for
networks with different M



Eigenvectors for Laplacian

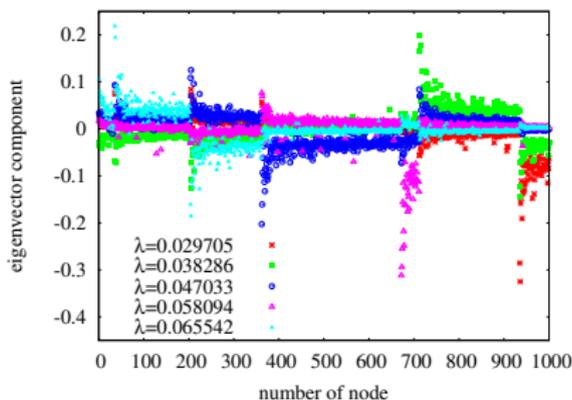
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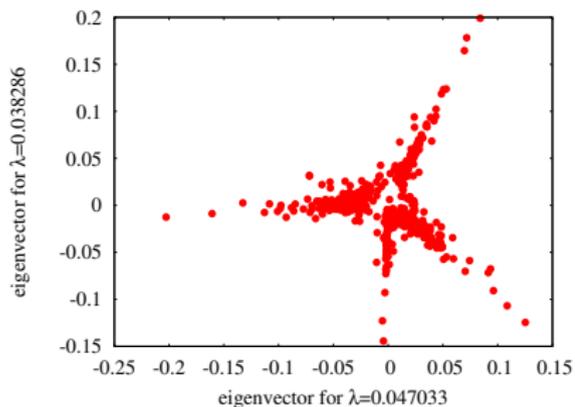
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Eigenvectors for G – 1 first
non-zero eigenvalues



Scatter plot of eigenvectors

Eigenvectors for Laplacian

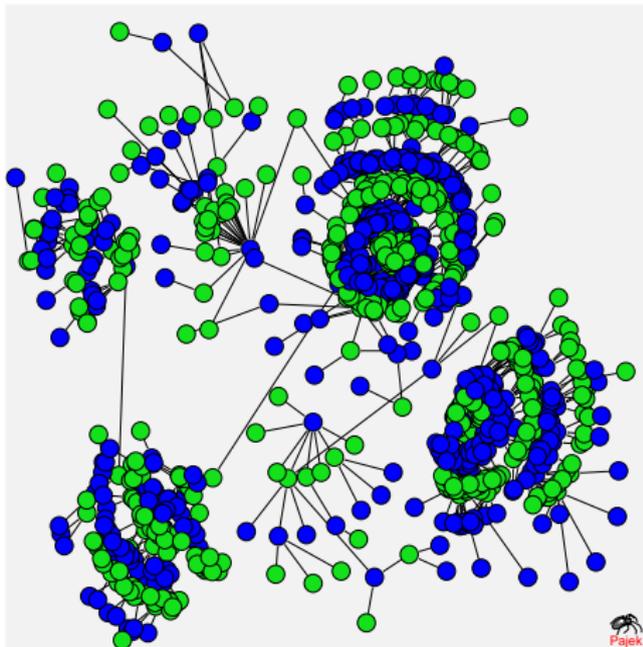
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Eigenvector for largest eigenvalue $\lambda = 2$
for tree of trees network.

- The largest eigenvalue in spectra is equal 2 only for tree networks ($M = 1, \alpha = 1$).
- Eigenvalues $\lambda = 1$ are related with chains.

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- We presented a model of multiscale networks and investigate spectral properties of adjacency matrix and normalized Laplacian.
- Spectral properties depend on global structure and structure of modules in networks.
- Eigenvector of adjacency matrix and Laplacian operator contain information about the structure of networks.

Acknowledgments

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- This work was done in collaboration with professor Bosiljka Tadić

